

**Lund Institute of Technology
Department of Building Acoustics**

**TRANSMISSION OF
STRUCTURE-BORNE
SOUND AT VARIOUS
TYPES OF JUNCTIONS
WITH THIN
ELASTIC LAYERS**

Steindor Gudmundsson

**Report TVBA-3016 · ISSN 0281-8477
Lund Sweden April 1984**



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PREFACE

The research presented in this report has been carried out at the department of Building Acoustics, Lund Institute of Technology.

The author is grateful to Professor Sven G Lindblad for introducing the project, and for his guidance and encouragement.

Mr Lars Persson has performed most of the acceleration level measurements, and Mr Robert Månsson has participated in some of the field measurements. I thank them both for their support and assistance.

Special thanks to Ms Kerstin Larsson, who efficiently prepared and typed the manuscript. I also wish to thank Mr Bo Zadig and Ms Ingbritt Larsson for their drawings, and Mr Charles Brand for his advise and corrections regarding questions of the English language.

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Lund, April, 1984

Steindor Gudmundsson

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NOTATION

B	- bending stiffness (Nm^2)
	- bending stiffness per unit width (Nm)
B	- index for bending wave
C	- spring constant proportional to moment displacements (Nm)
E	- Young's modulus (Pa)
F	- force (N)
	- force per unit width (N/m)
G	- shear modulus (Pa)
K	- spring stiffness (N/m)
	- spring stiffness per unit area (N/m^3)
K	- spring constant proportional to shear displacement (N/m)
K_x	- spring constant proportional to longitudinal displacements (N/m)
L	- length (m)
L	- index for longitudinal wave
L	- level (dB)
M	- bending moment (Nm)
	- bending moment per unit width (N)
M	- mass (kg)
P	- power (W)
R	- transmission loss (dB)
S	- area, cross sectional area (m^2)
T	- torsional stiffness (Nm^2)
Z_L	- longitudinal mechanical impedance (Ns/m)
b	- length (m)
b	- bandwidth (Hz)
c_B	- phase velocity of bending waves (m/s)
c_{Bg}	- group velocity of bending waves (m/s)
c_L	- phase velocity of longitudinal waves (m/s)
c_S	- phase velocity of shear waves (m/s)
c_T	- phase velocity of torsional waves (m/s)
e	- length (m)
f	- frequency (Hz)

f_A	- frequency of total attenuation (Hz)
f_T	- frequency of total transmission (Hz)
f_C	- critical frequency (Hz)
h, h_0	- height (m)
j	- the imaginary unit
k_B	- wavenumber of bending waves (m^{-1})
l	- length (m)
m	- mass per unit length (kg/m) - mass per unit area (kg/m^2)
m_0	- single mass (kg)
r	- coefficient for amplitude and phase of reflected bending wave
r_j	- coefficient for amplitude and phase of reflectional nearfield
r_S	- distance (to center of gravity) (m)
t	- time (s)
t	- coefficient for amplitude and phase of transmitted bending wave
t_j	- coefficient for amplitude and phase of transmissional nearfield
v	- velocity (m/s)
v_S	- velocity of the center of gravity (m/s)
w	- angular velocity (rad/s)
x, y, z	- cartesian coordinates
Δ	- difference
Θ	- mass moment of inertia ($kg\ m^2$) - mass moment of inertia per unit length ($kg\ m$)
Λ	- equivalent absorption length in a plate (m)
α	- help parameter ($\alpha = \omega^2 \Theta / (Bk_B)$) - help parameter ($\alpha = \omega^2 \Theta / (Bk_B \cdot (1 - (c_{T0} \sin \phi / c_B)^2))$)
β	- angle of rotation (rad)
β	- help parameter ($\beta = \omega c_B\ m/K$)
γ	- shear angle (rad)
γ	- help parameter ($\gamma = (\omega^2 / Bk_B) r_S^2 m_0 / (1 - j 2 Z_L / (\omega m_0))$)
γ_x	- help parameter ($\gamma_x = (\omega^2 / Bk_B) r_S^2 m_0 / (1 - j \frac{Z_L}{\omega m_0} (\frac{2K_x + j\omega Z_L}{K_x + j\omega Z_L}))$)

δ	- help parameter ($\delta = m_0 \omega / (m c_B)$)
	- help parameter ($\delta = m_0 \omega / (m c_B) \cdot (1 - (c_{B_0} \sin \phi / c_B)^4)$)
ϵ	- help parameter ($\epsilon = e \pi / \lambda_B$)
η	- loss factor
η	- displacement in the y-direction (m)
κ	- shear distribution parameter
κ	- help variable ($\kappa = k_{B_3} / k_{B_1}$)
λ_B	- wavelength of bending wave (m)
μ	- Poisson's ratio
ν	- help parameter ($\nu = B k_B / C$)
π	- 3.1415....
ρ	- density (kg/m^3)
ρ	- reflection efficiency
σ	- normal stress (Pa)
σ	- help parameter ($\sigma = \sqrt{1 + \sin^2 \phi}$)
σ_1	- help parameter ($\sigma_1 = 1 + (1 - \mu) \sin^2 \phi$)
τ	- shear stress (Pa)
τ	- transmission efficiency
ϕ	- angle of incidence (rad, deg)
ψ	- help parameter ($\psi = k_{B_3}^2 B_3 / (k_{B_1}^2 B_1)$)
ψ	- help parameter ($\psi = \sqrt{1 - \sin^2 \phi} = \cos \phi$)
ψ_1	- help parameter ($\psi_1 = 1 - (1 - \mu) \sin^2 \phi$)
ω	- angular frequency (rad/s)
ω_0	- angular resonance frequency (rad/s)

Some conventions

$\tilde{v}$	RMS value
$\hat{v}$	peak value
v_+	wave travelling in the + direction
v_-	wave travelling in the - direction
v_{-j}	nearfield decaying in the + direction
v_{-j}	nearfield decaying in the - direction
∇^2	Laplace operator (div grad)

INTRODUCTION

Methodics

The intention of this report is to deal with the structure-borne sound insulation capacity of a thin elastic layer, separating two structural elements. More specifically these structural elements are considered to be concrete slabs, resting on an elastic foundation.

This is a practical situation in many Swedish terrace houses without a cellar, built in lightweight building materials. Besides the elastic foundation (mineral wool, expanded plastic) the only structural coupling for the slabs is to the lightweight walls. Through the thin elastic interlayer the slabs are also coupled to their neighbouring slab, the floor of the next door apartment.

Because of this isolation from structural elements, that can effectively take up and transplant the energy of a primarily excited slab, the acoustic temperature of the slab becomes high. This in turn means that the transmission of sound to the neighbouring slab through the elastic interlayer becomes critical.

In Part 1 we consider the one-dimensional problem, which applies to beams or normal incidence in plates. Calculations are made both for idealized boundary conditions on each side of the interlayer, and then more practical boundary conditions are considered. If we have a lightweight wall at the junction, this may be modelled as an extra mass (or mass per length unit for a plate). The bending stiffness is only marginally increased.



We may also consider the case of a reinforcing beam at the junction. In the one dimensional case we have both an extra mass and an extra bending stiffness on both sides (or one side only) of the elastic interlayer. In practice the reinforcing beams are usually excentric, and calculations are made both for the symmetric and the excentric case.

In Part 2 we use a two-dimensional model, taking into account the oblique incidence of the bending waves in the plates. The elastic interlayer is considered to be locally reacting. Here we also consider several different boundary conditions. Primarily we consider the effect of a reinforcing beam or beams in combination with the elastic interlayer. Besides the extra mass and the extra bending stiffness we also have an extra torsional stiffness to consider. This makes an excentric beam difficult to model and we confine ourselves to cases where we have symmetric edge beams.

In Part 3 we publish an extensive series of measurements in a laboratory, where different types of interlayers and foundations are studied by measuring the transmission loss for the junction. We also consider the loss factors of the slabs, and discuss ways to increase the losses. Finally we make some comparisons between the theoretical solutions and the measurements. To make this comparison possible we have measured some of the material properties of the different interlayers. These measurements are also presented.

Historical review

The transmission and reflection of vibrational energy in structures at discontinuities was first analyzed in 1948 by Cremer [1]. The main results of this work are also published in [2].

Cremer considers beams and in some cases plates with a change in cross-section or material, with blocking masses, elastic interlayers and changes in direction at corners.

Cremer derives the amplitudes of the transmitted and reflected waves by considering an incident travelling bending wave and letting the waves fulfil the boundary conditions for the field variables at the discontinuity. Each boundary condition gives rise to an equation, and solving the sets of simultaneous equations gives the amplitudes of the different reflected or transmitted waves.

The two types of discontinuities we are mainly interested in, the elastic interlayer and the blocking mass, are treated by Cremer as follows: The elastic interlayers are considered in one dimension only, as a beam problem. In [1] and [2] calculations are shown, where the beams on each side of the interlayer are identical and both have the same direction. In [1] calculations are also presented for elastic interlayers at an L-shaped junction between two beams.

The symmetric blocking mass is treated in [1] and [2], both as an one-dimensional problem, and as a two-dimensional problem of a plate with a reinforcing beam.

In [2] the problem of an incident longitudinal wave in a beam with an excentric blocking mass is also treated. This produces reflected and transmitted bending waves as well as longitudinal waves.

The problem of an elastic interlayer in combination with a blocking mass at the same junction is not considered.

Exner and Böhme [3] (1953) investigated experimentally the bending wave transmission loss for elastic interlayers between duraluminium beams ($40 \times 40 \text{ mm}^2$ cross section). Their results are in good agreement with Cremer's theoretical predictions as long as the elastic layer is thin and has a low loss factor.

Kurtze, Tamm and Vogel [4] (1955) investigated experimentally L-shaped corners with and without elastic interlayers. They used beams made of plexiglass ($30 \times 30 \text{ mm}^2$ cross section), and the results confirmed Cremer's analysis in [1] with some minor complementations.

In [5] (1957) Müller presents an approximate analysis on the effect of an excentric blocking mass on a beam with an incident longitudinal wave. (The exact solution was later provided by Cremer in [2].)

Ungar [6] (1961) and Heckl [7] (1961) discuss the two-dimensional model for wave propagation in a plate reinforced with a beam. Ungar's results agree with the ones of Cremer in [1]. Heckl considers finite systems with one beam and a periodic array of beams respectively, and shows some comparisons with measurements which agree well with theory.

In [8] (1968) Skudrzyk treats longitudinally vibrating rods, mass-loaded on each side of a compliant element. He also discusses the bending wave

transmission of a mass-loaded beam joint.

Other interesting references, that treat the transmission and reflection of bending waves at a junction with an elastic interlayer or with a blocking mass have not been found.

On the other hand, many references can be found on the related problem of intersecting beams or plates. Some of these references have been studied too, as the calculation methods are similar.

Cremer produced the one-dimensional solution for L, T and + junctions in [1]. He also produced the two-dimensional solution for the L junction, which was however slightly corrected in the English edition of [2].

Mugiono [9] (1955) and Hinsch [10] (1960) compared experimental data with theoretical values for changes in the cross section and for intersecting beams. They used the same or similar theoretical methods as Cremer, and obtain a good agreement between theory and experiments.

Budrin and Nikiforov [11] (1964), carry on with similar theoretical methods for intersecting beams, and Zaborov [12] (1968) makes a simplified analysis neglecting longitudinal waves. In [13] (1970) he continues this analysis, and concludes that longitudinal wave transmission can be neglected in some cases, and simplified in other cases.

In [14] (1967) Kihlman solves the two-dimensional problem of four intersecting semi-infinite plates. He produces analytical expressions for different transmission coefficients as functions of the angle

of incidence and calculates the diffuse values. Through a power flow model, comparisons are made between theory and measurements. Further experimental investigations are reported by Kihlman in [15] (1970).

In [16] (1971) Bhattacharya, Mulholland and Crocker consider two T-junctions one after the other. Only perpendicular incidence with longitudinal and flexural waves is considered. This problem is also dealt with by Bhattacharya [17] (1970), and there he makes the two-dimensional analysis as well. He presents the analytical equations, but no practical examples are shown as "the computer programs for these computations are complex and too long".

In [18] (1974) Gibbs uses similar methods to Cremer and Kihlman and investigates transmission at the junctions of infinite plates for several plate configurations. He also uses power flow methods and presents experimental results from quarter scale models.

In [19] (1976) Gibbs continues his work in cooperation with C L S Gilford. They present calculations of several types of junctions, where the transmission coefficient is shown as a function of the angle and the frequency. The calculations are still similar to the ones of Kihlman and Cremer, but the analytical expressions are not solved explicitly, but presented in a matrix describing a set of simultaneous equations. The matrix equation is then solved numerically for different configurations. Power flow methods are used to compare theory to measurements in a quarter scale model.

In [20] (1977) Gibbs and Gilford study the effect of damping on the vibrational energy of concrete plates, which form a T-junction.

In [21] (1980) Davies and Gibbs study an L-junction, and compare measured and predicted angular variation of the acceleration level difference across the junction. A similar study of a T-junction is made by the same authors in [22] (1981).

In [23] (1980) Craik discusses coupling loss factors for plates with different type of joints (T, L, +). He assumes no displacement in either plate plane, so no longitudinal or transverse waves are generated. He states that this does not have a serious effect on the coupling loss factor.

Coupling loss factors for different types of joints are also considered in [24] and [25] (1981) by Wöhle, Beckmann and Schreckenbach. They use methods similar to Cremer and Kihlman and include longitudinal and transverse waves in their calculations.

Finally we may mention [26] and [27] (1981) where Craven and Gibbs present a "new approach" for solving sound transmission and mode couplings at junctions of thin plates. This new approach lies in a powerful computer program, where each wave can be represented with a displacement vector and a wave vector. All wave types can then be treated uniformly instead of separate considerations for the six possible cases.

PART 1

ONE DIMENSIONAL
THEORETICAL STUDY
(BEAMS)

1.1 Introduction

In the first part of this report we shall deal with the one-dimensional problem of structure-borne sound at junctions with thin elastic layers. First we define the theoretical model, and then we consider our first type of junction, which has no extra constraints, just two beams connected by an elastic interlayer. We consider first the case of identical beams, and then we investigate the influence of losses in the interlayer, and the influence of two different beams.

Secondly we consider the idealized boundary conditions or extra constraints at the end of each beam (on each side of the interlayer). These boundary conditions are "simply supported" and "guided" ends respectively.

Thirdly we consider the effects of a blocking mass on a beam. We consider primarily a symmetric ideal blocking mass and then we investigate the influence of a finite clamping length and the excentricity of the blocking mass.

Finally we combine the elastic interlayer and different blocking masses at a junction. We consider cases where only an extra mass is added, which is assumed to represent the case of a lightweight wall at the junction. We also consider the case where both an extra mass and an extra bending stiffness has to be taken into account. This represents the case of a reinforcing beam, and by an additional extra mass we have the case of a lightweight wall and a reinforcing beam. These reinforcing beams are usually excentric in practice, so we show calculations both for the symmetric and the excentric case.

At the end of Part one we have a short summary with concluding remarks.

1.2 Theoretical model

The theoretical model used here is the same model as the one used by Cremer in [1], [2]. This means that only pure bending waves are considered, thus neglecting rotatory inertia and shear forces. The beams are assumed to be semi-infinite, so the involved waves are always free and progressive.

1.2.1 Longitudinal waves

In some cases longitudinal waves are considered.



For a beam, with contraction permitted in two directions, we have quasi-longitudinal waves with the propagation velocity

$$c_L = \sqrt{E/\rho} \quad (1)$$

If we consider plates (and normal incidence), contraction is permitted in one direction only and we have quasi-longitudinal waves with the propagation velocity

$$c_L = \sqrt{E/(\rho(1 - \mu^2))} \quad (2)$$

Quasi-longitudinal waves in a beam are represented by two field variables. Here we choose to use the velocity, v_x , and the force, F_x .

The wave equation to be fulfilled is:

$$\frac{\partial^2}{\partial x^2}(F_x, v_x) = \left(\frac{1}{c_L}\right)^2 \frac{\partial^2}{\partial t^2}(F_x, v_x) \quad (3)$$

The longitudinal mechanical impedance is

$$Z_L = F_x/v_x = c_L \cdot m \quad (4)$$

where m is the mass per unit length for a beam.

The number of required boundary conditions is two, one for each field variable.

1.2.2 Bending waves

The most important wave type is of course the bending wave. Here we only consider pure bending waves and thus confine ourselves to frequencies for which $\lambda_B \geq 6h$.

Bending waves must be represented by four field variables instead of two.

Here, the following four variables are chosen:

- i) the transverse velocity, v_y
- ii) the angular velocity, w_z
- iii) the bending moment, M_z
- iv) the shear force, F_y

It is customary in acoustics to choose the algebraic signs of these field variables somewhat different to those conventionally chosen in statics. (In statics it is furthermore preferred to work with transverse displacement (η) and rotational angle (β) where:

$$\beta = \frac{\partial \eta}{\partial x} \quad , \quad w_z = \frac{\partial \beta}{\partial t} \quad , \quad v_y = \frac{\partial \eta}{\partial t} \quad) \quad (5)$$

Cremer [1], [2] motivates the acoustical choice of signs by first selecting v_y positive in the positive y-direction and w_z positive for counter-clockwise rotation, and then choosing M_z and F_y in such a way that the products $M_z \cdot w_z$ and $F_y \cdot v_y$ both represent power flows in the x-direction. This sign convention is used here, see fig 1.2.1.

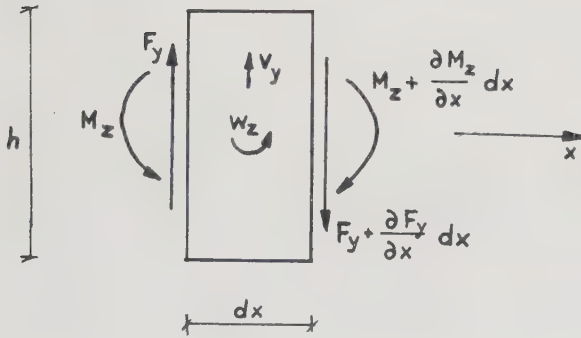


Fig 1.2.1. Positive directions on a beam element in a bending wave.

The four field variables are related by four differential equations, which couple the variables cyclically:

$$w_z = \frac{\partial v_y}{\partial x} \quad (6)$$

$$\frac{\partial M_z}{\partial x} = -B \frac{\partial w_z}{\partial x} \quad (7)$$

$$F_y = -\frac{\partial M_z}{\partial x} \quad (8)$$

$$-\frac{\partial F_y}{\partial x} = m \cdot \frac{\partial v_y}{\partial t} \quad (9)$$

From these equations the partial differential equation for one-dimensional bending waves becomes:

$$-B \frac{\partial^4 (v_y, w_z, M_z, F_y)}{\partial x^4} = m \frac{\partial^2 (v_y, w_z, M_z, F_y)}{\partial t^2} \quad (10)$$

If we assume a sinusoidal bending wave, written in v_y :

$$v_y = \hat{v}_{y0} \cdot e^{j(\omega t - k_B x)} \quad (11)$$

we get after insertion in (10):

$$k_B^4 = \omega^2 m / B \quad (12)$$

$$k_B = \begin{cases} \pm \sqrt{\omega} \sqrt[4]{m/B} \\ \pm j \sqrt{\omega} \sqrt[4]{m/B} \end{cases} \quad (13)$$

The real solutions of k_B give propagating waves and the imaginary solutions give exponentially decaying nearfields.

The general solution thus consists of four parts, corresponding to the fourth order of the differential equation:

$$v_y = \hat{v}_+ e^{-jk_B x} + \hat{v}_- e^{+jk_B x} + \hat{v}_{-j} e^{-k_B x} + \hat{v}_j e^{+k_B x} \quad (14)$$

(Time dependence omitted).

Real k_B gives a bending wave phase velocity $c_B = \omega / k_B$:

$$c_B = \sqrt{\omega} \sqrt[4]{B/m} \quad (15)$$

The group velocity, with which the energy propagates is however

$$c_{Bg} = \frac{d\omega}{dk_B} = 2\sqrt{B/m} \cdot k_B = 2c_B \quad (16)$$

1.2.3 Transmission and Reflection at Discontinuities

The problem can be analyzed by separating any time-dependence into sinusoidal components and treating each such component separately.

We assume an incident wave arriving from $x < 0$ represented by

$$v_{1+} e^{-jk_{B1} x} \quad (17)$$

In the region $x < 0$ this incident wave results in a reflected wave

$$v_{1-} e^{+jk_{B1}x} = r v_{1+} e^{+jk_{B1}x} \quad (18)$$

where r accounts for changes in both phase and amplitude. We also have a decaying nearfield:

$$v_{1j} e^{+k_{B1}x} = r_j v_{1+} e^{+k_{B1}x} \quad (19)$$

The resulting velocity in the region $x < 0$ is thus given by

$$v_1(x) = v_{1+} (e^{-jk_{B1}x} + r e^{+jk_{B1}x} + r_j e^{+k_{B1}x}) \quad (20)$$

Similarly the field behind the discontinuity consists of a transmitted wave and a nearfield:

$$v_2(x) = v_{1+} (t e^{-jk_{B2}x} + t_j e^{-k_{B2}x}) \quad (21)$$

We have 4 unknowns, r , r_j , t , t_j , whose values are determined by the 4 boundary conditions, one for each field variable.

1.3 Elastic interlayer between two beams

1.3.1 Two identical beams

(This case is analyzed by Cremer in [1], [2].)

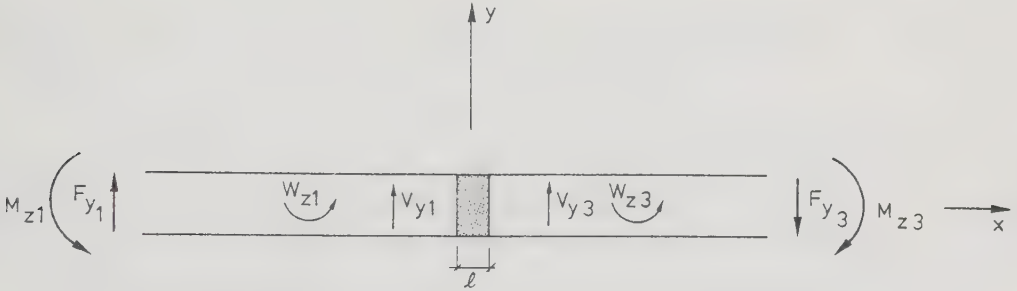


Fig 1.3.1. The configuration used in the calculations.

The elastic interlayer is assumed to be thin, so that it may be considered as a pure spring. Two boundary conditions are obtained by assuming that the moments and shear forces on each side of the elastic layer must be equal. In obtaining the other two boundary conditions, the deformations of the elastic layer are considered to be proportional to the shear forces and the moments:

$$F_y = K \cdot \Delta \eta \quad (1)$$

$$M_z = C \cdot \Delta \beta \quad (2)$$

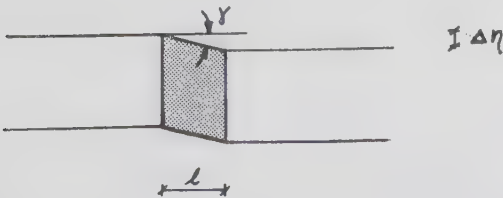


Fig 1.3.2. Shear displacement of the elastic layer.

The shear stress τ can be written as:

$$\tau = G_2 \cdot \gamma = G_2 \cdot \Delta\eta/l \quad (3)$$

$$\tau = F_y \cdot \kappa / S_2 \quad (4)$$

Together (3) and (4) give:

$$F_y = \frac{S_2 G_2}{\kappa \cdot l} \Delta\eta \quad (5)$$

where κ accounts for the shear distribution.

For rectangular cross sections in beams we have $\kappa = 1.2$, but this value is probably not directly applicable to the thin layer considered here.

However κ takes values close to 1 except for extreme cross sections like I-beams, so assuming $\kappa \approx 1$ will probably not cause any major errors here.

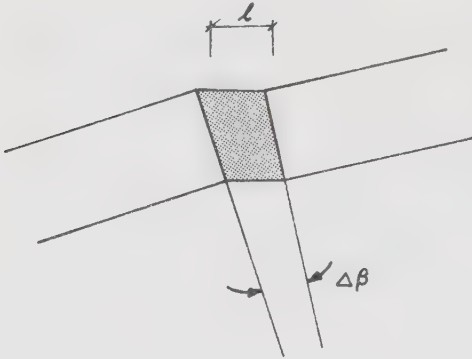


Fig 1.3.3. Moment displacement of the elastic layer.

The normal stress σ_x varies across the section as:

$$\sigma_x(y) = E_2 \cdot \frac{y}{l} \Delta\beta \quad (6)$$

$$M_z = \int_S \sigma_x y \, dS = \frac{B_2 \Delta\beta}{l} \quad (7)$$

The spring constants from equations (1), (2) can now be written as:

$$K = \frac{G_2 S_2}{\kappa l} \approx \frac{G_2 S_2}{l} \quad (8)$$

$$C = \frac{B_2}{l} \quad (9)$$

The four boundary conditions for the configuration in fig 1.3.1 are then:

- i) $M_{Z_1} = M_{Z_3}$
- ii) $F_{Y_1} = F_{Y_3}$
- iii) $(v_{Y_1} - v_{Y_3})/j\omega = F_{Y_3}/K$
- iv) $(w_{Z_1} - w_{Z_3})/j\omega = M_{Z_3}/C$

Now we use expressions (1.2.20) and (1.2.21) for the wave fields on each side of the discontinuity, and write all four field variables in terms of the transverse velocity v_y by using the cyclic equations (1.2.6-9).

After this the four boundary conditions above take the following form, when the beams on each side are assumed to be identical:

- i) $[-1 - r + r_j] = [-t + t_j]$
- ii) $[-j - jr + r_j] = [jt - t_j]$
- iii) $[1 + r + r_j] - [t + t_j] = \frac{\omega C_B^m}{K} [jt - t_j]$
- iv) $[-j + jr + r_j] + [jt + t_j] = \frac{Bk_B}{C} [t - t_j]$

We now introduce two parameters

$$\beta = \frac{\omega C_B^m}{K} \quad (10)$$

$$v = \frac{Bk_B}{C} \quad (11)$$

[Note that v is chosen the same as Cremer's, but β in his notation is $\epsilon^2 v^3$]

The equation system can now be written as:

$$\begin{bmatrix} -1 & 1 & 1 & -1 \\ -j & 1 & -j & 1 \\ 1 & 1 & (-1-j\beta) & (\beta-1) \\ j & 1 & (j-v) & (1+v) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} 1 \\ -j \\ -1 \\ +j \end{bmatrix} \quad (12)$$

This matrix equation can of course easily be solved numerically for different values of v and β . However the analytical solutions of the variables can often give some valuable information, which is not so easily seen in the above matrix expression nor easily deduced from various numerical examples.

The analytical expressions here are:

$$r = \frac{-4v + 4\beta + 2v\beta}{(4v + 4\beta) + j(-16 - 4v + 4\beta + 2v\beta)} \quad (13)$$

$$t = \frac{j(-16 - 4v + 4\beta)}{(4v + 4\beta) + j(-16 - 4v + 4\beta + 2v\beta)} \quad (14)$$

We are mainly interested in the transmission and reflection efficiencies, and the two field variables above are sufficient for the calculation of these. The expressions for r_j and t_j are therefore omitted here.

The transmission and reflection efficiencies are defined on an energy basis, and the power in a bending wave follows $P_B = 2c_B \cdot m \cdot \tilde{v}_B^2$. We have then the transmission efficiency:

$$\tau = \frac{2c_B \cdot m \cdot \tilde{v}_{3+}^2}{2c_B \cdot m \cdot \tilde{v}_{1+}^2} = |t|^2 \quad (15)$$

and the reflection efficiency:

$$\rho = \frac{2c_B \cdot m \cdot \tilde{v}_{1-}^2}{2c_B \cdot m \cdot \tilde{v}_{1+}^2} = |r|^2 \quad (16)$$

If there are no losses, we also have from the laws of conservation of energy that $\tau + \rho = 1$ so it is sufficient to calculate only one of the variables r and t . However calculating τ and ρ from two different analytical expressions and making sure that $\tau + \rho = 1$ gives an opportunity to find errors in the expressions themselves or in the programming work.

From the expressions for r and t , we see that a total transmission occurs when r vanishes:

$$4\beta + 2\nu\beta = 4\nu \quad (17)$$

and a total attenuation occurs when t vanishes:

$$4\beta = 4\nu + 16 \quad (18)$$

The equation for total transmission may be written as:

$$\beta(2 + \nu) = 2\nu \quad (19)$$

For most practical cases we have $2 \ll \nu = B/C \cdot k_B$ ($k_B = (2\pi/c) \cdot \sqrt{f \cdot f_c} > 1$ for practical plates in the interesting frequency region, and if the elastic layer is to have an appreciable effect it should have $C \ll B$). The condition for total transmission is therefore

$$\beta \approx 2 \quad (20)$$

or

$$f_T \approx \frac{4650}{f_c} (K/B)^{2/3} \quad (21)$$

If we assume that the ratio E_2/G_2 is close to 3 in most cases (constrained E-modulus for thin layers is somewhat higher than the actual value, and μ probably has a value somewhere between 0.3 and 0.5 for practical elastic interlayers) we may write:

$$f_T \approx \frac{12000}{f_c} \left(\frac{E_2/l}{E_1 \cdot h^2} \right)^{2/3} \quad (22)$$

(We also assume $h_1 = h_2 = h_3 = h$).

The equation for total attenuation may be written as

$$\beta = \nu + 4 \quad (23)$$

As in the case of total transmission we assume $\nu \gg 4$. (This assumption is even more in its right here as the frequency f_A is shown to be higher than f_T .) We therefore have the condition for total attenuation

$$\beta \approx \nu \quad (24)$$

or

$$f_A \approx \frac{2900}{f_C} \text{ K/C} \quad (25)$$

If we again assume that E_2/G_2 is close to 3 and $h_1 = h_2 = h_3 = h$, we may write:

$$f_A \approx \frac{12000}{f_C \cdot h^2} \quad (26)$$

The frequency of total transmission f_T is often found to lie within the interesting frequency range for building acoustics, while f_A in many cases takes values higher than that, and often exceeds the frequency limit of applicability for pure bending waves.

If we take the expression for t and simplify it by making the same approximations as before ($\nu \gg 4$) we may write:

$$t = \frac{j(-4\nu + 4\beta)}{(4\nu + 4\beta) + j(-4\nu + 4\beta + 2\nu\beta)} \quad (27)$$

and if we now confine ourselves to frequencies "sufficiently" far below f_A (about 3 octaves lower) β can be considered small compared to ν and we have:

$$t = \frac{-j4\nu}{4\nu + j(-4\nu + 2\nu\beta)} \quad (28)$$

$$\tau = |t|^2 = \frac{1}{1 + (1 - \beta/2)^2} \quad (29)$$

$$R = -10 \log \tau = 10 \log \left(1 + \left(\frac{2 - \beta}{2} \right)^2 \right) \text{ dB} \quad (30)$$

As $\beta \sim k_B^3$ we have for $f_T < f \ll f_A$ that R increases as $30 \log f$, and we have $R = 0$ for $\beta = 2$. (Observe a typing error in [2] in the approximate equation above: $1 - \epsilon^2 v^3$ should be $2 - \epsilon^2 v^3$.)

Cremer uses the above approximate expression as a theoretical curve in his figures in [2] and this is also used in [3] for comparison with experimental values. This gives a rise in R towards lower frequencies below f_T , which is misleading.

Below f_T , β has become so small ($\beta = 0.35$ one octave below f_T) that the term $v\beta/2$ is negligible compared to v . We have then

$$t \approx \frac{-jv}{v - jv} \quad (31)$$

$$\tau \approx 0.5 \quad \text{or } R = 3 \text{ dB} \quad (32)$$

At still lower frequencies we can no longer neglect the factor 4 compared to v and $t \rightarrow 1$ or $R \rightarrow 0$.

As a practical example for calculations we take one of the cases studied in [28], [29]:

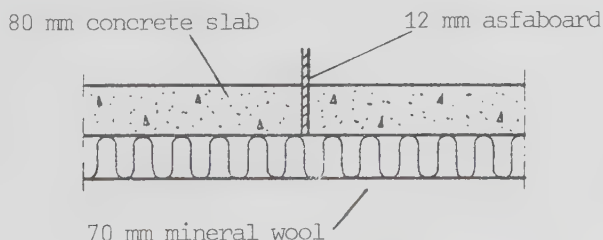


Figure 1.3.4. Configuration of the practical example (from [28], [29]).

No correction is at this point made for the influence that the elastic foundation has on the bending waves in the concrete slabs. The so called "asfboard" is

an asphalt impregnated porous fibre board. Its shear modulus has been measured [30] as $G = 5 \cdot 10^6 \text{ N/m}^2$ and its (constrained) E-modulus as $E = 1.8 \cdot 10^7 \text{ N/m}^2$. The ratio E_2/G_2 is thus slightly higher than previously estimated (3.6 compared to 3).

Below we show the curves for τ , ρ calculated by expressions (15) and (16) in one diagram, and then separately the transmission loss ($-10 \log \tau$), and the reflection loss ($-10 \log \rho$).

The frequency of total transmission is approximately given by (22):

$$f_T \approx \frac{12000}{225} \left(\frac{1.8 \cdot 10^7 / 0.012}{2.6 \cdot 10^{10} \cdot 0.08^2} \right)^{2/3} = 230 \text{ Hz}$$

or more accurately according to (21)

$$f_T \approx 200 \text{ Hz}$$

The approximate expression is seen to give a fairly good accuracy here. The true frequency of total transmission is around 180 - 190 Hz according to the diagrams below.

The frequency of total attenuation is approximately

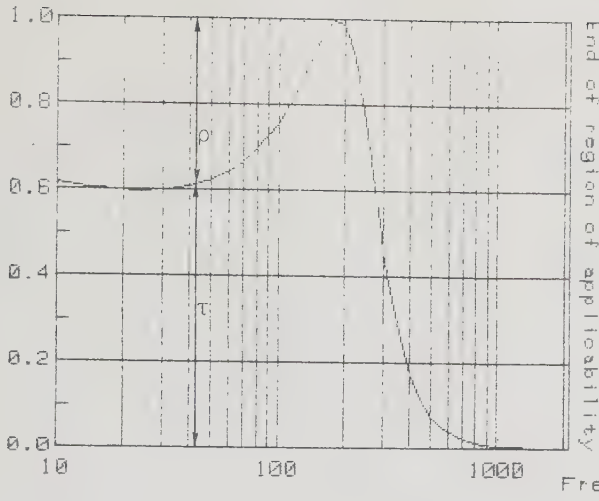
$$f_A \approx \frac{12000}{225} \cdot \left(\frac{1}{0.08} \right)^2 = 8000 \text{ Hz}$$

(uncertain value, as it lies beyond the limit of applicability.)

The form of the curve is as predicted: $R = 0$ at f_T , then reaching a nearly constant (slightly decreasing) value of about 2 dB below f_T , and above f_T increasing as $30 \log f$, (9 dB/octave).

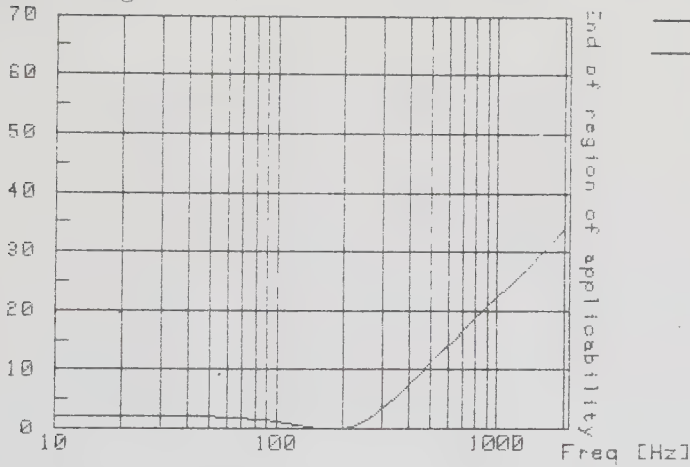
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	$E \text{ [Pa]}$	$2.6\text{E}+10$	$1.8\text{E}+07$	$2.6\text{E}+10$
G-modulus	$G \text{ [Pa]}$		$5.0\text{E}+06$	
Density	$\text{Rho}[\text{kg/m}^3]$	2300	800	2300
Poiss. ratio	$\text{My} [1]$	0.25	0.30	0.25
Thickness	$H \text{ [m]}$	0.080	0.080	0.080
Width of layer	$L \text{ [m]}$		0.012	

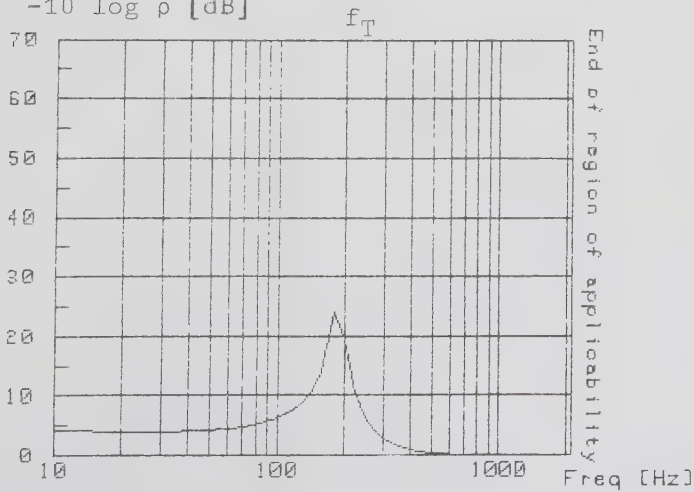


Transmission Efficiency for a Bending Wave at a Junction with an Elastic Layer [E].

$-10 \log \tau$ [dB]



$-10 \log \rho$ [dB]



1.3.2. Loss in the Elastic Interlayer

The internal damping of the interlayer can be taken into account by writing Young's modulus in a complex form as: $E_2(1 + j\eta_2)$, where η_2 is the loss factor of the material.

In the previous chapter, we would then get:

$$K = K' + jK'' = \frac{G_2 S_2}{\kappa l} (1 + j\eta_2) \quad (33)$$

$$C = C' + jC'' = \frac{B_2}{l} (1 + j\eta_2) \quad (34)$$

which in turn would give:

$$\beta = \beta' + j\beta'' \approx \frac{\omega C_{B_3} m_3}{K'} (1 - j\eta_2) \quad (35)$$

$$v = v' + jv'' \approx \frac{B_3 k_{B_3}}{C'} (1 - j\eta_2) \quad (36)$$

We can use the same formulas as before with β and v replaced by the complex ones in eq (35) and (36). This would have very little influence on τ and ρ , except near f_T .

If we denote the dissipated part of energy by δ we now have:

$$\tau + \rho + \delta = 1 \quad (37)$$

If we confine ourselves to frequencies close to f_T , we can make similar simplifications as in part 1.3.1. This gives

$$\delta \approx \frac{4\eta\beta}{(2 + \eta(2\beta - 2))^2 + (\delta - 2)^2} \quad (38)$$

which has its maximum for

$$\beta = 2 \sqrt{\frac{2(1 - \eta + \eta^2)}{1 + 4\eta^2}} \quad (39)$$

or

$$\beta \approx 2\sqrt{2} \quad (40)$$

This corresponds to a frequency of about one 1/3 octave step above f_T .

For low frequencies we have the asymptote

$$\delta \approx \eta\beta/2 \quad (41)$$

and for high frequencies

$$\delta \approx 4\eta/\beta \quad (42)$$

so we see that δ decreases on both sides of the maximum.

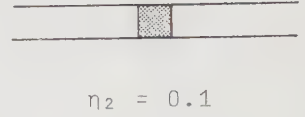
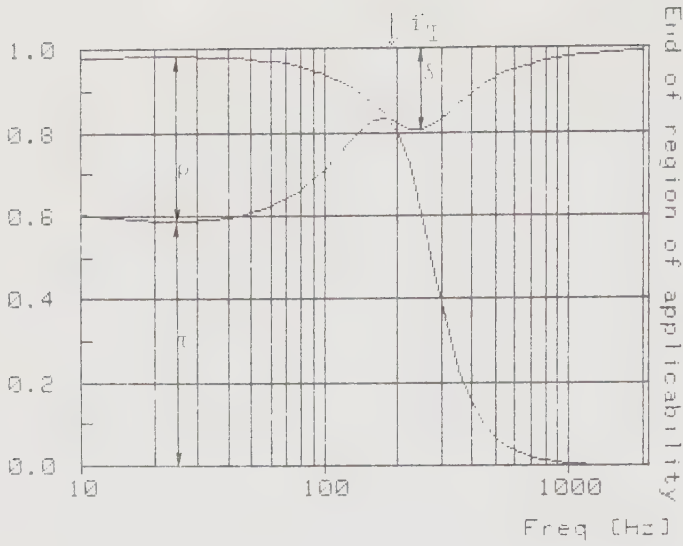
As for the influence of η on the transmission loss, we can write with the same approximations once again:

$$\tau \approx \frac{1 + \eta^2}{[1 + \eta(\beta - 1)]^2 + [1 - \beta/2 + \eta]^2} \quad (43)$$

$$R = -10 \log \tau \approx 10 \log (1 + \eta\beta + (\frac{2-\beta}{2})^2) \quad (44)$$

At f_T we have $\beta \approx 2$ and $R \approx 10 \log (1 + 2\eta)$ instead of $R = 0$. If we take $\eta = 10 \%$ as an example, we get $R \approx 1$ dB instead of 0 dB.

For frequencies somewhat higher or lower than f_T , there is very little influence on τ (or R). See the figure below, which shows the same example as before, but with $\eta = 10 \%$.



Transmission Efficiency for a Bending Wave at a Junction with an Elastic Layer. The Dissipation and the Reflection Efficiency is also shown.

1.3.3 Elastic interlayer between two different beams

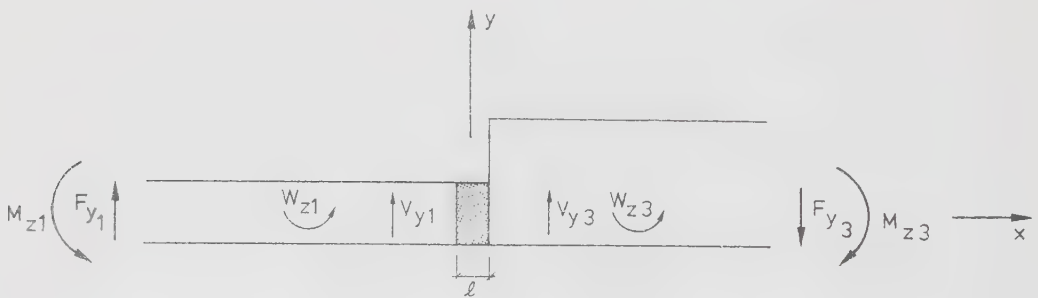


Figure 1.3.7. The beams have bending stiffnesses and masses, that are not necessarily identical. In the figure, this is exemplified by different heights of the beams.

The boundary conditions are the same as in part 1.3.1:

- i) $M_{z_1} = M_{z_3}$
- ii) $F_{y_1} = F_{y_3}$
- iii) $(v_{y_1} - v_{y_3})/j\omega = F_{y_3}/K$
- iv) $(w_{z_1} - w_{z_3})/j\omega = M_{z_3}/C$

However, when we write the field variables in terms of v_y , we cannot simplify by assuming $m_1 = m_3$ and $B_1 = B_3$. Instead we get:

- i) $[-1-r-r_j] = \frac{B_3 k_{B_3}^2}{B_1 k_{B_1}^2} [-t+t_j]$
- ii) $[+j-jr+r_j] = \frac{B_3 k_{B_3}^3}{B_1 k_{B_1}^3} [jt - t_j]$
- iii) $[1+r+r_j] - [t+t_j] = \frac{\omega C_{B_3} m_3}{K} [jt - t_j]$
- iv) $\frac{k_{B_1}}{K_{B_3}} [-j+jr+r_j] + [jt+t_j] = \frac{B_3 k_{B_3}}{C} [t-t_j]$

It is convenient to introduce some help-parameters here again. We choose first as a parameter the ratio in equation i):

$$\psi = \frac{B_3 k_{B_3}^2}{B_1 k_{B_1}^2} = \sqrt{\frac{m_3 B_3}{m_1 B_1}} \quad (45)$$

It then becomes most simple to choose the second parameter as the ratio of the bending wavelengths:

$$\kappa = \frac{\lambda_{B_1}}{\lambda_{B_3}} = \frac{k_{B_3}}{k_{B_1}} = \sqrt[4]{\frac{m_3 B_1}{m_1 B_3}} \quad (46)$$

[Cremer uses these same parameters in [2] when discussing the case of "changes in materials and cross-sections" (part V.1)].

The parameters v and β must here be related specifically to one of the beams, and we have:

$$\beta_3 = \frac{\omega c B_3 m_3}{K} \quad (47)$$

$$v_3 = \frac{B_3 k_{B_3}}{C} \quad (48)$$

The equation system can now be written as:

$$\begin{bmatrix} -1 & +1 & +\psi & -\psi \\ -j & +1 & -j\kappa\psi & +\kappa\psi \\ +1 & +1 & -(j\beta_3+1) & (\beta_3-1) \\ +j & +1 & \kappa(j-v_3) & \kappa(1+v_3) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -j \\ -1 \\ +j \end{bmatrix} \quad (49)$$

This provides the following analytical expressions for r and t :

$$r = \frac{\{-v_3(\kappa+\kappa^2\psi)+\beta_3(\kappa+\psi)+v_3\beta_3\kappa-\kappa(\psi-1)^2\}+j\{v_3(\kappa^2\psi-\kappa)+\beta_3(\psi-\kappa)+2\psi(\kappa^2-1)\}}{\{v_3(\kappa+\kappa^2\psi)+\beta_3(\kappa+\psi)\}+j\{-\kappa(\psi+1)^2-2\psi(\kappa^2+1)-v_3(\kappa+\kappa^2\psi)+\beta_3(\kappa+\psi)+v_3\beta_3\kappa\}} \quad (50)$$

$$t = \frac{j\{-2v_3\kappa+2\beta_3-2(\kappa+1)(\psi+1)\}}{\{v_3(\kappa+\kappa^2\psi)+\beta_3(\kappa+\psi)\}+j\{-\kappa(\psi+1)^2-2\psi(\kappa^2+1)-v_3(\kappa+\kappa^2\psi)+\beta_3(\kappa+\psi)+v_3\beta_3\kappa\}} \quad (51)$$

We see that for identical beams ($\psi = \kappa = 1$), these expressions become the same as those in part 1.3.1.

The transmission and reflection efficiencies are obtained in the same way as in part 1.3.1:

$$\tau = \frac{\gamma_{c_{B_3}} \cdot m_3 \cdot \hat{V}_3^2}{\gamma_{c_{B_1}} \cdot m_1 \cdot \hat{V}_1^2} = \kappa\psi |t|^2 \quad (52)$$

$$\rho = \frac{\gamma_{c_{B_1}} \cdot m_1 \cdot \hat{V}_1^2}{\gamma_{c_{B_3}} \cdot m_3 \cdot \hat{V}_3^2} = |r|^2 \quad (53)$$

If there are no losses, we also have $\tau + \rho = 1$.

The minimum transmission loss occurs when the numerator of r has its minimum. With similar simplifications as before (in part 1.3.1) we now get:

$$\beta_{3T} \approx (1 + \kappa\psi) \quad (54)$$

As $\kappa\psi = 1$ for identical beams, we have for this case $\beta \approx 2$ as in part 1.3.1.

As an example, we take the case where we double the height of the beam on the transmitting side, but keep the elastic interlayer unchanged. When both beams are identical, we have

$$\beta = \beta_0 = 2 \quad (55)$$

but with the height doubled on the transmitting side we have:

$$\beta_{3T} = (1 + 4/\sqrt{2}) \quad (56)$$

This corresponds to the following condition written with the help of β_0 for identical beams (as we have $\beta_3 = (\kappa\psi)\beta_0$):

$$\beta_0 = (1 + \sqrt{2}/4) = 1.35 \quad (57)$$

An identical result is obtained by doubling the height on the incident side. Thus changes in material or cross section of the beams always lower the frequency of maximum transmission, if the elastic layer is kept unchanged.

The frequency of total attenuation is also lowered as the approximate condition for this phenomenon is:

$$\beta_{3A} \approx \kappa v_{3A} \quad (58)$$

which gives us for doubling of the height:

$$\beta_{3A} \approx v_{3A}/\sqrt{2}$$

If we use v_0 and β_0 for identical beams, we also have (as $\beta_3 = (\kappa\psi)\beta_0$ and $v_3 = (\psi/\kappa)v_0$):

$$\beta_0 \approx v_0/\sqrt{2} \quad (60)$$

For the case taken here as an example, a doubling of the height, only changes β_T from 2 to 1.35 which corresponds to little more than one 1/3-octave step (30 % decrease in f_T). f_A is reduced by about 41 %. In a simplified form we get:

$$\tau = \kappa\psi |t|^2 \approx \frac{4\kappa\psi}{(1 + \kappa\psi)^2 + ((1 + \kappa\psi) - \beta_3)^2} \quad (61)$$

or

$$R \approx 10 \log \left(\frac{(1 + \kappa\psi)^2}{2\kappa\psi} - \frac{1 + \kappa\psi}{2\kappa\psi} \beta_3 + \frac{\beta_3^2}{4\kappa\psi} \right) \quad (62)$$

R has its minimum at $\beta_3 = (1 + \kappa\psi)$, and for our case of doubled height we have

$$R \approx 10 \log(2.6 - 0.677 \beta_3 + 0.0884 \beta_3^2) \quad (63)$$

$$R_{\min} = 1 \text{ dB} \quad (64)$$

R increases as $30 \log f$ for high frequencies, and the curve lies $10 \log(\kappa\psi) = 4.5 \text{ dB}$ higher than the one for identical beams.

If losses in the elastic layer are included here, as well we have

$$\delta \approx \frac{4\eta\kappa\psi\beta_3}{[(1 + \kappa\psi) + \eta(2\beta - (1 + \kappa\psi))]^2 + [\beta - (1 + \kappa\psi)]^2} \quad (65)$$

which gives us $\delta_{\max}$ at:

$$\beta_3 = (1 + \kappa\psi) \sqrt{\frac{1(1 - \eta + \eta^2)}{1 + 4\eta^2}} \quad (66)$$

or

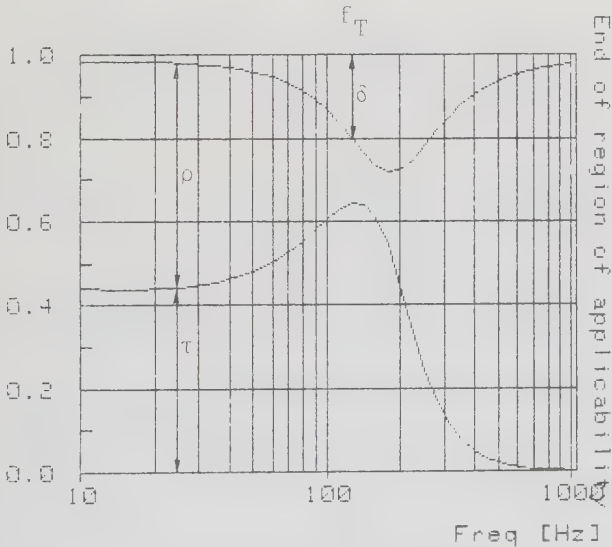
$$\beta_3 \approx (1 + \kappa\psi) \sqrt{2} \quad (67)$$

As in the case of identical beams this corresponds to a frequency one 1/3-octave step above f_T . The approximate expression for the transmission loss now becomes:

$$R \approx 10 \log \left(\frac{(1+\kappa\psi)^2}{4\kappa\psi} + \frac{\eta\beta_3(1+\kappa\psi)}{2\kappa\psi} + \frac{(\beta_3-(1+\kappa\psi))^2}{4\kappa\psi} \right) \quad (68)$$

At the frequency of maximum transmission, $\beta_3 \approx (1 + \kappa\psi)$, R will be increased by $10 \log (1 + 2\eta)$, which amounts to about 1 dB for $\eta = 10 \%$. For our example of doubling the height, we would thus get $R_{\min} = 2$ dB instead of 1 dB, when the losses are not taken into account.

Below we show the same example as in part 1.3.2, except that the height of the beam on the transmitting side has now been doubled. This means a more narrow frequency region of applicability, and the frequency f_T is lowered from approximately 180 Hz to approximately 140 Hz. $\delta_{\max}$ occurs here at approximately 180 Hz.



$$h_2 = 2h_1$$

$$\eta_2 = 0.1$$

Transmission Efficiency for a Bending Wave at a Junction with an Elastic Layer. The Dissipation and the Reflection Efficiency is also shown.

1.4 Idealized boundary conditions for beams separated with an elastic interlayer

1.4.1 Elastic interlayer between two identical beams with one end guided

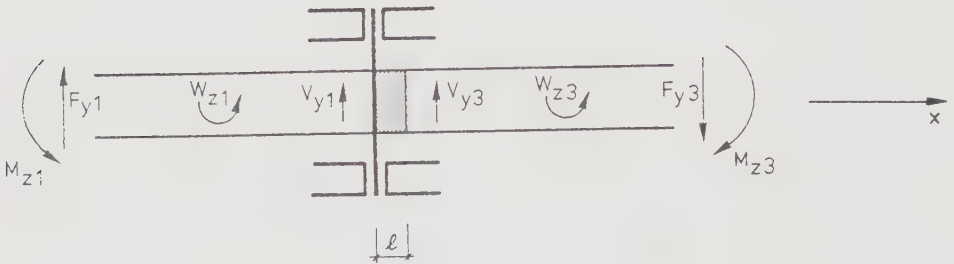


Fig 1.4.1. The beam with the guided end is free to displace laterally, but the guided end is prevented from rotating.

We make the same assumptions as before about the elastic layer, and use the same parameters ν and β .

Here the four boundary conditions are:

- i) $w_{z1} = 0$
- ii) $F_{y1} = F_{y3}$
- iii) $(v_{y1} - v_{y3})/j\omega = F_{y3}/K$
- iv) $-w_{z3}/j\omega = M_{z3}/C$

which results in the following equation system:

$$\begin{bmatrix} j & 1 & 0 & 0 \\ -j & 1 & -j & 1 \\ 1 & 1 & (-1-j\beta) & (-1+\beta) \\ 0 & 0 & (+j-\nu) & (1+\nu) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} j \\ -j \\ -1 \\ 0 \end{bmatrix} \quad (1)$$

The analytical expressions for r and t are:

$$r = \frac{(-4 + 4\beta - 2\nu + 2\nu\beta) + j(-4\nu + 2\nu\beta)}{(-4 + 4\beta + 2\nu\beta) + j(-4 - 6\nu + 2\nu\beta)} \quad (2)$$

$$t = \frac{-j(4 + 4\nu)}{(-4 + 4\beta + 2\nu\beta) + j(-4 - 6\nu + 2\nu\beta)} \quad (3)$$

If we make similar simplifications as in part 1.3.1, we see that the numerator of $\rho = |r|^2$ takes the form

$$(-2\nu + 2\nu\beta)^2 + (-4\nu + 2\nu\beta)^2 \quad (4)$$

which cannot become zero, but has its minimum for $\beta = 1.5$. We therefore have no frequency of total transmission, but maximum transmission occurs at a frequency about 1/3 octave lower than the previous f_T (1.3.20). The value of $\tau_{\max}$ is about $1 - 2/18 = 0.9$.

The numerator of τ cannot become zero either, so there is no frequency of total attenuation, f_A . The increase in τ with the frequency is however very similar to the one in part 1.3.1 at least as long as $f \ll f_A$:

$$R = -10 \log \tau = 10 \log \left(\frac{2\beta^2 - 6\beta + 9}{4} \right) \quad (5)$$

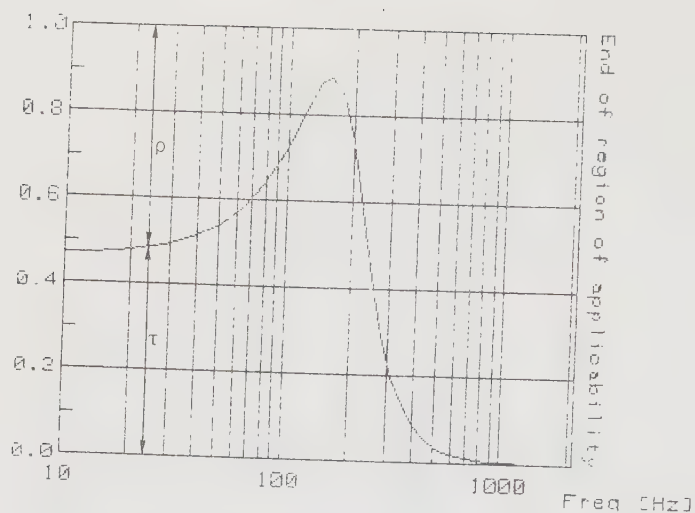
The asymptote has the same slope as before:

$R \approx 10 \log (\beta^2/2)$ but lies 3 dB higher.

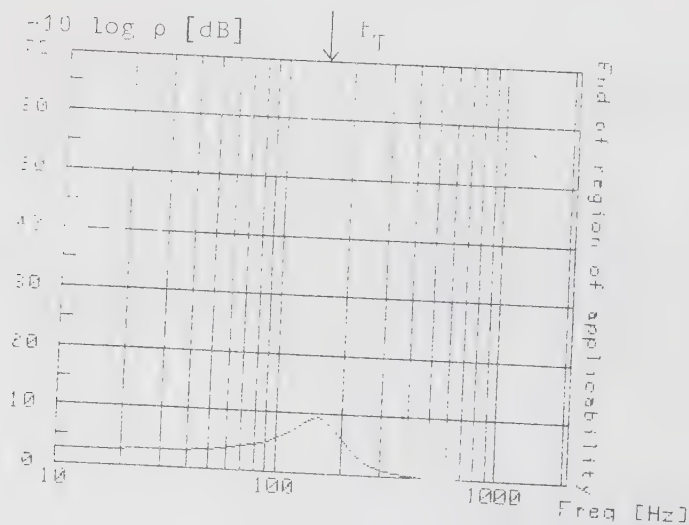
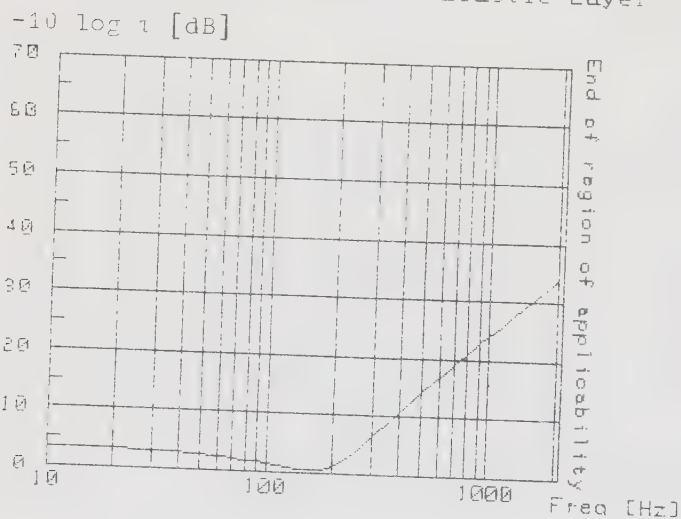
Below are the calculations for the same practical example as before, but with one end guided.

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m ³]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	



Transmission Efficiency for a Bending Wave at a Junction with an Elastic Layer [Eg]



If the other end is chosen as the guided one, the same expressions for r and t are achieved, except that the imaginary part in the numerator of r changes its sign. We therefore get the same r and ρ , as is to be expected for reasons of symmetry.

1.4.2 Elastic interlayer between two identical beams with both ends guided

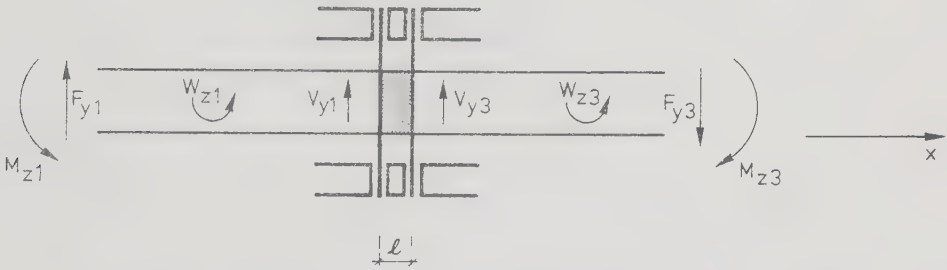


Fig 1.4.2. The beams are free to displace laterally, but the guided ends are prevented from rotating.

With the same assumptions as before, we get the following four boundary conditions

- i) $w_{z1} = 0$
- ii) $F_{y1} = F_{y3}$
- iii) $(v_{y1} - v_{y3})/j\omega = F_{y3}/K$
- iv) $w_{z3} = 0$

which results in the equation system:

$$\begin{bmatrix} j & 1 & 0 & 0 \\ -j & 1 & -j & 1 \\ 1 & 1 & (-1-j\beta) & (-1+\beta) \\ 0 & 0 & j & 1 \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} j \\ -j \\ -1 \\ 0 \end{bmatrix} \quad (6)$$

and the analytical expressions for r and t :

$$r = \frac{-4 + 4\beta}{-4 + 4\beta - 4j} \quad (7)$$

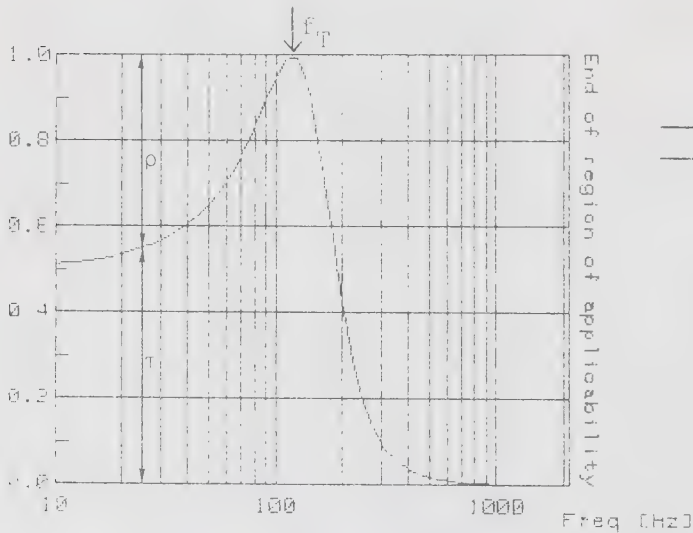
$$t = \frac{-4j}{-4 + 4\beta - 4j} \quad (8)$$

Now we again have a frequency of total transmission, when the numerator of r becomes zero, and this occurs when $\beta = 1$, which means a frequency about two 1/3-octave steps lower than the original f_T (1.3.20). No frequency of total attenuation exists, but the increase in τ with frequency is similar to the previous one.

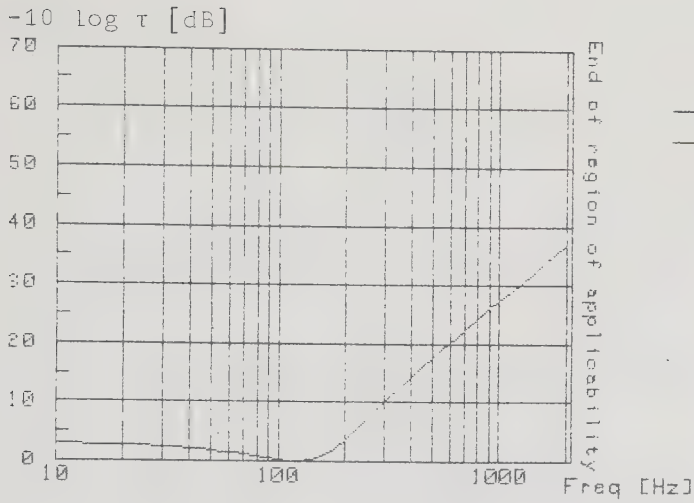
$$R = -10 \log \tau = 10 \log (1 + (\beta - 1)^2) \quad (9)$$

The curve lies 6 dB above the original curve and has the same slope as long as $f \ll f_A$.

Below are the calculations for the same practical example as before with both ends guided.



Transmission Efficiency for a Bending Wave
at a Junction with an Elastic Layer [Egg]



1.4.3 Elastic interlayer between two identical beams with one end simply supported

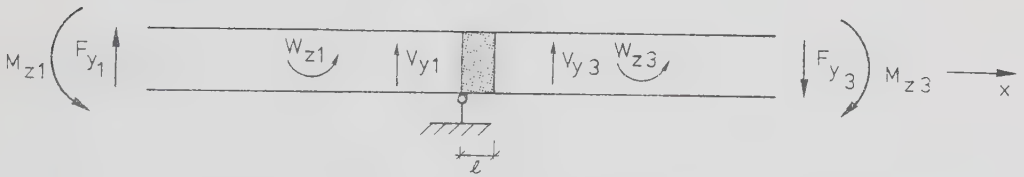


Fig 1.4.3. The beam with the simply supported end is free to rotate but prevented from deflecting laterally.

With the same assumptions as before we get the following four boundary conditions:

- i) $M_{z1} = M_{z3}$
- ii) $v_{y1} = 0$
- iii) $-v_{y3}/j\omega = F_{y3}$
- iv) $(w_{z1} - w_{z3})/j\omega = M_{z3}/C$

and the equation system:

$$\begin{bmatrix} -1 & +1 & +1 & -1 \\ +1 & +1 & 0 & 0 \\ 0 & 0 & -(j\beta+1) & (\beta-1) \\ +j & +1 & (j-v) & (1+v) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -1 \\ 0 \\ +j \end{bmatrix} \quad (10)$$

which results in

$$r = \frac{(-4 - 4v + 2\beta + 2v\beta) - j(4\beta + 2v\beta)}{(4 + 4v - 2v\beta) + j(-4 + 6\beta + 2v\beta)} \quad (11)$$

$$t = \frac{j(-4 + 4\beta)}{(4 + 4v - 2v\beta) + j(-4 + 6\beta + 2v\beta)} \quad (12)$$

If we make the same approximations as before we seem to have a minimum in $\rho (= |r|^2)$, when its simplified numerator has a minimum:

$$(-4v + 2v\beta)^2 + (2v\beta)^2 \quad (13)$$

which is for $\beta = 1$.

However, the denominator assumes this same value for $\beta = 1$, so we have a maximum instead: $\rho = 1$.

This is confirmed by the expression for t , which is zero for $\beta = 1$.

We have therefore a frequency of total attenuation f_A , and it is the same one as the frequency of total transmission in 1.4.2!

There is no frequency of total transmission here, and the expression for the transmission loss is:

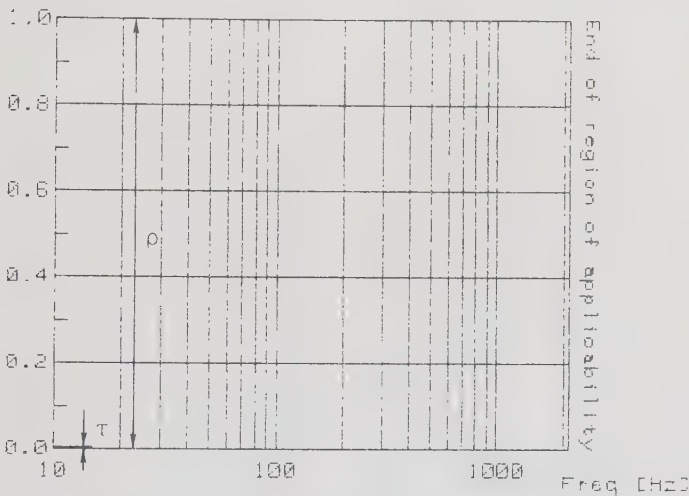
$$R = -10 \log \tau = 10 \log \left(\frac{v^2 \frac{1 - \beta + \beta^2/2}{(1 - \beta)^2}} \right) \quad (14)$$

For $f \ll f_A$ we have $\beta \ll 1$ and $R = 20 \log v$

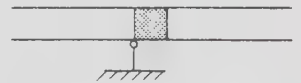
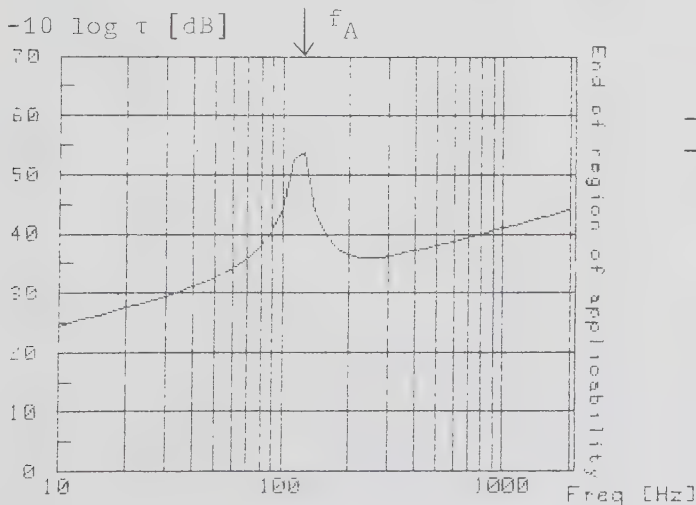
For $f \gg f_A$ we have $\beta \gg 1$ and $R = 10 \log (v^2/2)$

Thus we have a curve with the slope $10 \log f$, and a frequency of total attenuation f_A which typically lies within the frequency region of building acoustics. The curve above f_A is displaced 3 dB downwards compared to the curve below f_A .

Below we show the same example once again. Only $-10 \log \tau$ is shown as $\rho \approx 1$ and $-10 \log \rho \approx 0$ for all frequencies.



Transmission Efficiency for a Bending Wave at a Junction with an Elastic Layer. [Eo]



1.4.4 Elastic interlayer between two identical beams
with both ends simply supported

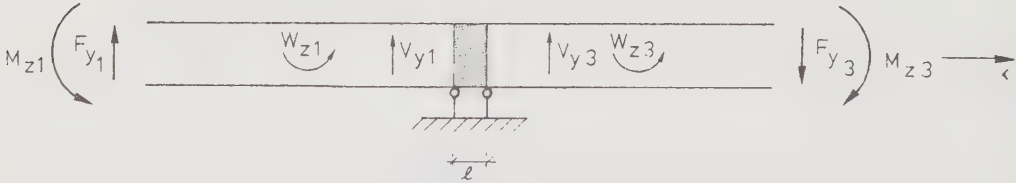


Fig 1.4.4. The beams are free to rotate, but prevented from deflecting laterally.

The same assumptions as previously defined give the following four boundary conditions:

- i) $M_{z1} = M_{z3}$
- ii) $v_{y1} = 0$
- iii) $v_{y3} = 0$
- iv) $(w_{z1} - w_{z3})/j\omega = M_{z3}/C$

and the equation system:

$$\begin{bmatrix} -1 & +1 & +1 & -1 \\ +1 & +1 & 0 & 0 \\ 0 & 0 & +1 & +1 \\ +j & +1 & (j-\nu) & (1+\nu) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -1 \\ 0 \\ +j \end{bmatrix} \quad (15)$$

which results in:

$$r = \frac{4 + 4\nu}{-4 - 4\nu + 4j} \quad (16)$$

$$t = \frac{4j}{-4 - 4\nu + 4j} \quad (17)$$

Here we have

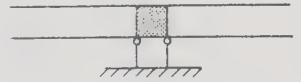
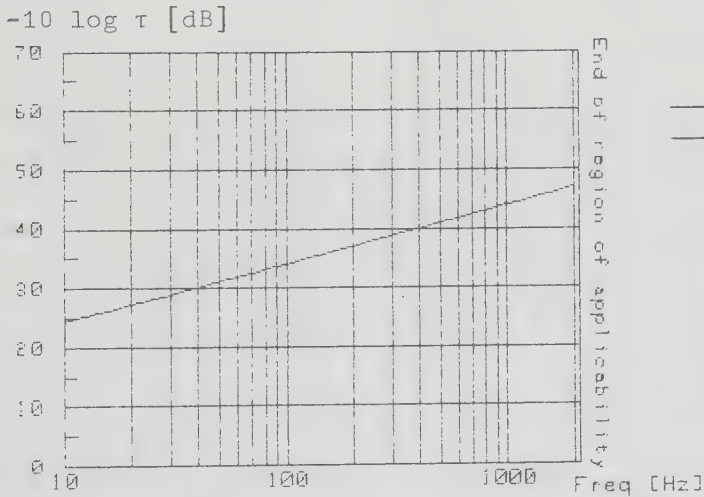
$$R = -10 \log \tau = 10 \log (1 + (1 + v)^2) \quad (18)$$

or

$$R \approx 20 \log v \quad (19)$$

which is the same as for the case where only one end is simply supported, not counting the total attenuation at f_A and the 3 dB jump.

Below we show $-10 \log \tau$ for the same practical example as before.



1.5 Blocking masses

1.5.1 Symmetric blocking mass on a beam

(This case is analyzed by Cremer in [1], [2].)

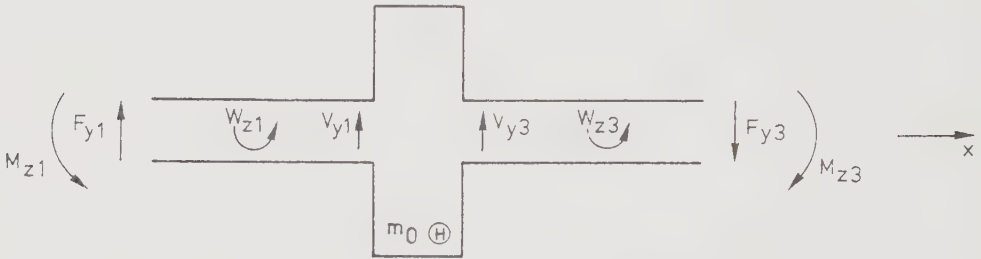


Fig 1.5.1 The blocking mass is assumed to be rigidly attached to the beam and has the mass m_0 and the mass moment of inertia Θ .

For a rectangular cross section (height $\times$ width = $h_0 \times b_0$) we have per unit of length:

$$m_0 = \rho \cdot b_0 \cdot h_0 \quad [\text{kg/m}] \quad (1)$$

$$\Theta = \rho (b_0 h_0^3 + h_0 b_0^3) / 12 \quad [\text{kg m}] \quad (2)$$

Here, however we chose to think of m_0 and Θ as something added to the beam and m_0 was corrected accordingly

$$m_0 = \rho \cdot b_0 (h_0 - h) \quad (3)$$

The correction for Θ would in most cases only be marginal, so this was omitted and expression (2) used.

Two boundary conditions are obtained by assuming that the deformations (here represented by v_y and w_z) are equal on each side of the blocking mass.

In obtaining the other two boundary conditions, the forces and moments are assumed to be different on each side of the blocking mass, and this difference accelerates the mass:

$$\begin{aligned} \text{i)} \quad M_{Z_1} - M_{Z_3} &= j\omega\theta w_{Z_3} \\ \text{ii)} \quad F_{Y_1} - F_{Y_3} &= j\omega m_0 v_{Y_3} \\ \text{iii)} \quad v_{Y_1} &= v_{Y_3} \quad (= v_{Y_2}) \\ \text{iv)} \quad w_{Z_1} &= w_{Z_3} \quad (= w_{Z_2}) \end{aligned}$$

With a similar procedure as the one used before we now get with identical beams on each side:

$$\begin{aligned} \text{i)} \quad [-1 - r + r_j] - [-t + t_j] &= \frac{\omega^2\theta}{Bk_B} [-jt - t_j] \\ \text{ii)} \quad [j - jr + r_j] - [jt - t_j] &= \frac{\omega m_0}{mc_B} [t + t_j] \\ \text{iii)} \quad [1 + r + r_j] &= [t + t_j] \\ \text{iv)} \quad [-j + jr + r_j] &= [-jt - t_j] \end{aligned}$$

We now introduce two additional parameters:

$$\alpha = \frac{\omega^2\theta}{Bk_B} \quad (4)$$

$$\delta = \frac{\omega m_0}{mc_B} \quad (5)$$

[Note that δ is chosen in the same way as Cremer's μ but α in his notation is $v^2\mu^3$]

The equation system is then:

$$\begin{bmatrix} -1 & +1 & (1+j\alpha) & (\alpha-1) \\ -j & +1 & (\delta-j) & (\delta+1) \\ +1 & +1 & -1 & -1 \\ j & +1 & +j & +1 \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -j \\ -1 \\ +j \end{bmatrix} \quad (6)$$

which gives us

$$r = \frac{-4\delta + 4\alpha + 2\alpha\delta}{(4\delta + 4\alpha) - j(16 + 4\delta - 4\alpha - 2\alpha\delta)} \quad (7)$$

$$t = \frac{-j(16 + 4\delta - 4\alpha)}{(4\delta + 4\alpha) - j(16 + 4\delta - 4\alpha - 2\alpha\delta)} \quad (8)$$

Similar to the case of an elastic layer there exists a frequency of total transmission when the numerator of t equals zero:

$$4\delta = 4\alpha + 2\alpha\delta \quad (9)$$

Total attenuation occurs when the numerator of t equals zero:

$$16 + 4\delta = 4\alpha \quad (10)$$

Unfortunately the variables α and δ are of the same order of magnitude for many practical cases, and furthermore they do not differ much from the values of the numerical constants. Any simplifications are therefore more difficult here.

However as α and δ increase with the frequency, we may in some cases be able to neglect the numerical constant at f_A and estimate the frequency region in which f_A lies:

$$\alpha_A \approx \delta_A \quad (11)$$

$$f_A \approx \frac{3000}{f_c} \frac{m_0}{\theta} \quad (12)$$

We can also write

$$\delta_T/\delta_A \approx (1 + \delta_T/2) \alpha_T/\alpha_A \quad (13)$$

or

$$f_A/f_T \approx (1 + \delta_T/2) \quad (14)$$

which in many practical cases means that f_T lies between one and two octaves below f_A .

The above formulas are only intended to indicate roughly the frequency regions of f_A and f_T . For more exact evaluation formulas (9) and (10) must be used.

Since f_T and f_A are relatively close together, we have a steeply increasing transmission loss curve from f_T to f_A .

Below f_T , t and τ soon assume values close to 1 and we have $R \approx 0$.

Above f_A , the factors containing α soon take over and

$$\tau = |t|^2 \approx \frac{(4\alpha)^2}{(4\alpha)^2 + (4\alpha + 2\alpha\delta)^2} \quad (15)$$

or

$$R = -10 \log \tau = 10 \log (1 + (1 + \delta/2)^2) \quad (16)$$

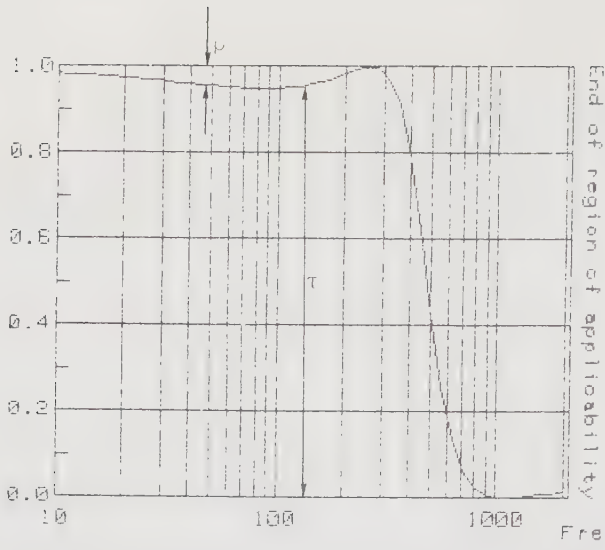
which means a slowly increasing curve as $10 \log f$.

Below we show as a practical example the same slab as in the previous examples ($h = 0.08$ m) with a blocking mass ($b_0 \times h_0 = 2h \times 5h$).

The approximate formulas for f_A and f_T give here $f_A \approx 700$ Hz and f_T ca 200 - 350 Hz. From the diagrams we read that $f_A \approx 1100$ Hz and $f_T \approx 250$ Hz.

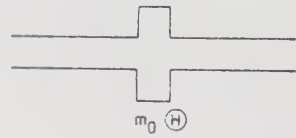
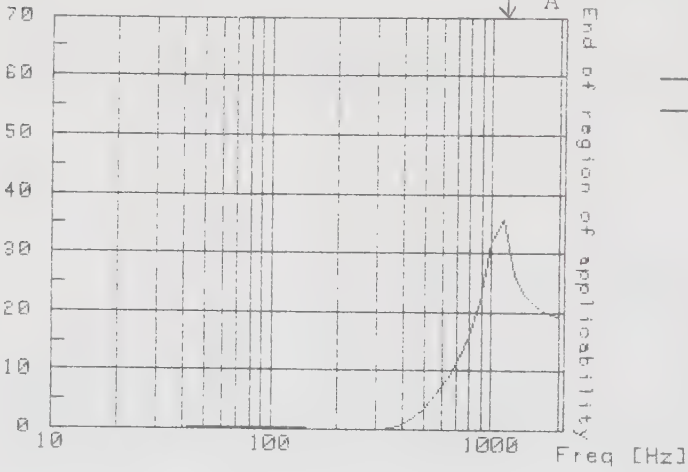
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10		2.6E+10
Density	Rho[kg/m3]	2300		2300
Poiss. ratio	My [I]	0.25		0.25
Thickness	H [m]	0.080		0.080
Dimensions of Block-				
ing Mass	Bo x Ho [m]		0.160 x 0.400	
Extra mass	Mo [kg/m]		118	
MassMom of Inert	[kgm]		2.3E+00	

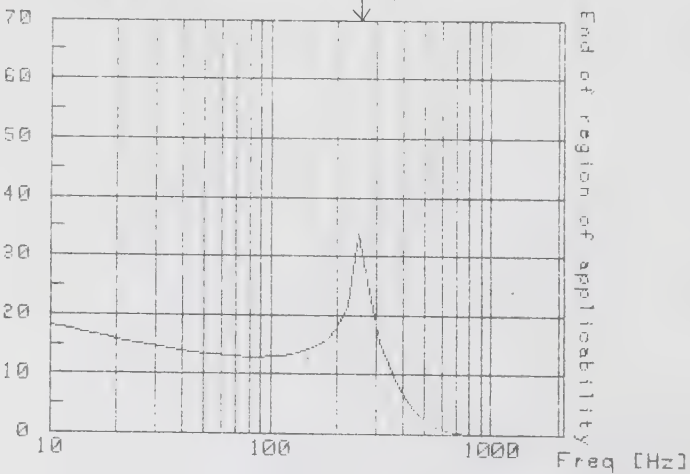


Transmission Efficiency for a Bending Wave at a Junction with a Symmetric Blocking Mass [Bx]

$-10 \log \tau$ [dB]



$-10 \log \rho$ [dB]



In [1], [2] some attention is paid to the case where the blocking mass may not be rigidly connected to the beam, and thus would have an angular velocity different to that of the beam ($\omega_2 \neq \omega_3$).

The blocking mass would then have a natural frequency ω_0 of rotational vibrations, high for rigid connections, low for more elastic connections.

It is shown that it is sufficient to exchange α for $\alpha/(1 - (\omega/\omega_0)^2)$ to account for the elastic connection.

If ω_0 is sufficiently high above the frequencies f_T and f_A , no significant change in the transmission loss curve will occur.

If ω_0 lies in the same frequency region as f_T and f_A for a rigid connection, f_A will be shifted towards lower frequencies.

Above ω_0 we would also have a decreasing α with an increasing frequency, so above f_A the transmission loss slowly decreases:

$$\tau = |t|^2 \approx \frac{(16 + 4\delta)^2}{(4\delta)^2 + (16 + 4\delta)^2} \quad (17)$$

$$R = -10 \log \tau \approx 10 \log \left(1 + \frac{\delta^2}{(4 + \delta)^2} \right) \quad (18)$$

with the limiting value 3 dB.

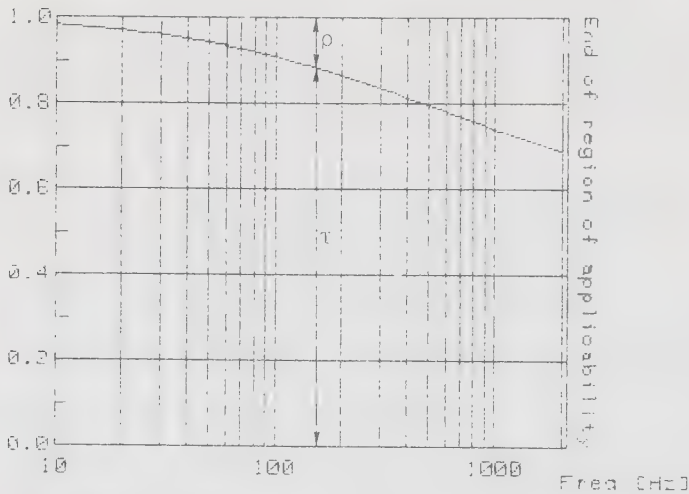
If ω_0 lies sufficiently far below the frequency region of f_T and f_A for rigid connection, we have (as the terms containing α can be neglected) that eq(18) is valid for the entire frequency region. We have no frequencies of total attenuation or total transmission, only slowly increasing values of R ranging from 0 to the limiting value 3 dB.

In the practical cases considered in this report, we will mostly deal with concrete slabs with concrete beams as blocking masses, where the slabs and beams are cast in one piece.

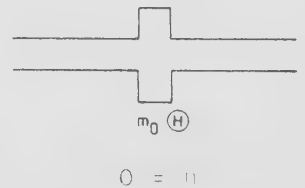
This connection is assumed to be sufficiently rigid as to give values of ω_0 well above the frequency region of interest, especially as the practical θ -values for these constructions are relatively low. Therefore the theory for rigid connection is used hereafter.

On the other hand, we may have situations, where the blocking mass could almost be considered as hinged ($\alpha = 0$). An example of this is a lightweight wall standing on our concrete slab. The wall is assumed to be sufficiently well fastened to the slab, so that it follows its lateral movements, at least at moderate frequencies. The fastening is however not stiff enough to take up the moments.

Below we show the same example as previously for rigid connection but now with $\alpha = 0$.



$-10 \log \tau$ [dB]



1.5.2 Symmetric blocking mass with a finite length of clamping

(This case is analyzed by Cremer in [1])

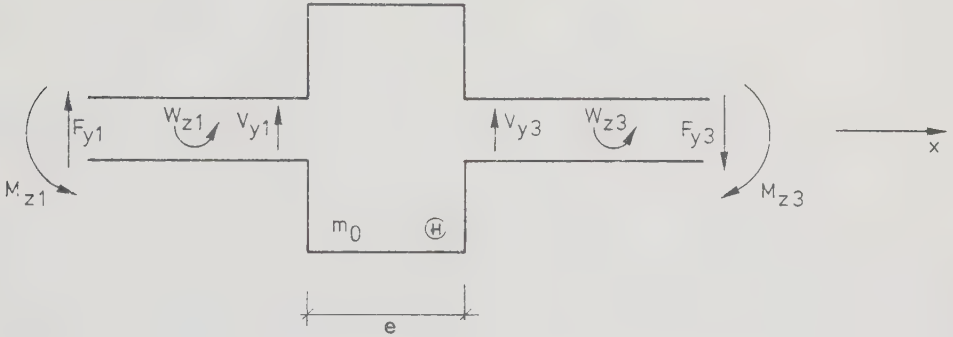


Fig 1.5.2. The blocking mass is assumed to be rigidly attached to the beam and has the mass m_0 and the mass moment of inertia Θ . The finite clamping length, e , is included in the boundary conditions.

The first three boundary conditions from 1.5.1 must be complemented with correction terms, taking into account the finite length of clamping.

- i) $M_{z1} - M_{z3} - (F_{y1} + F_{y3}) e/2 = j\omega\Theta w_{z3}$
- ii) $F_{y1} - F_{y3} = j\omega m_0 (v_{y1} + v_{y3})/2 = j\omega m_0 (v_{y3} - w_{z3} e/2)$
- iii) $v_{y1} = v_{y3} - ew_{z3}$
- iv) $w_{z1} = w_{z3} (= w_{z2})$

This gives with the same procedure as before:

$$i) \quad [-1 - r + r_j] - [-t + t_j] + \frac{e\pi}{\lambda_B} [j - jr + r_j + jt - t_j] = \frac{\omega^2 \Theta}{Bk_B} [-jt - t_j]$$

$$ii) \quad [j - jr + r_j] - [jt - t_j] = \frac{m_0 \omega}{m c_B} \{[-t - t_j] - \frac{e\pi}{\lambda_B} [jt + t_j]\}$$

$$\text{iii)} \quad [1 + r + r_j] = [t + t_j] - ek_B[-jt - t_j]$$

$$\text{iv)} \quad [-j + jr + r_j] = [-jt - t_j]$$

We now introduce the additional parameter

$$\epsilon = \frac{e\pi}{\lambda_B} \quad (19)$$

[In Cremer's notation: $\epsilon = \sigma e\mu/(2r)$]

The equation system here is now:

$$\begin{bmatrix} -(1+j\epsilon) & (1+\epsilon) & (1+j(\epsilon+\alpha)) & -(1+\epsilon-\alpha) \\ -j & +1 & (\delta+j(\epsilon\delta-1)) & (1+\delta+\epsilon\delta) \\ +1 & +1 & -(1+j2\epsilon) & -(1+2\epsilon) \\ +j & +1 & +j & +1 \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} 1-j\epsilon \\ -j \\ -1 \\ j \end{bmatrix} \quad (20)$$

which gives

$$r = \frac{-4\delta + 4\alpha + 2\alpha\delta + 16\epsilon + \epsilon^2(16 + 4\delta)}{4\delta + 4\alpha + \epsilon(16 + 8\delta) + \epsilon^2(16 + 4\delta) - j(16 + 4\delta - 4\alpha - 2\alpha\delta + 16\epsilon - 4\epsilon^2\delta)} \quad (21)$$

$$t = \frac{-j(16 + 4\delta - 4\alpha + \epsilon(16 + 8\delta) + 4\epsilon^2\delta)}{4\delta + 4\alpha + \epsilon(16 + 8\delta) + \epsilon^2(16 + 4\delta) - j(16 + 4\delta - 4\alpha - 2\alpha\delta + 16\epsilon - 4\epsilon^2\delta)} \quad (22)$$

If we let $e \rightarrow 0$ then $\epsilon \rightarrow 0$ and equations (21) and (22) reduce to those in 1.5.1 where the clamping length e was not taken into consideration.

In 1.5.1 the frequency of total attenuation was given by

$$\alpha = 4 + \delta \quad (23)$$

Now this condition becomes:

$$\alpha = 4 + \delta + \epsilon(4 + 2\delta) + \epsilon^2\delta \quad (24)$$

which means a somewhat higher frequency of total attenuation, f_A . In fact if e is sufficiently large, there is no such frequency since $\epsilon^2\delta$ will then always be larger than α :

$$k_B^3 \cdot \left(\frac{e}{2}\right)^2 \frac{m_0}{m} = \epsilon^2 \delta > \alpha = k_B^3 \frac{\theta}{m} \quad (25)$$

or

$$e > 2\sqrt{\frac{\theta}{m_0}} \quad (26)$$

In our practical example from 1.5.1 we have $e = 0.16$ m and $2\sqrt{\theta/m_0} = 0.28$ m.

The frequency of total transmission is also affected and the condition becomes

$$2\delta = \alpha(2 + \delta) + 8\epsilon + \epsilon^2(8 + 2\delta) \quad (27)$$

As ϵ is small for low frequencies, the last term can be neglected in many practical cases, but the term 8ϵ cannot, and this means a somewhat lower frequency of total transmission.

Here we also have the possibility that no frequency of total transmission will exist, which occurs if

$$4\epsilon > \delta \quad (28)$$

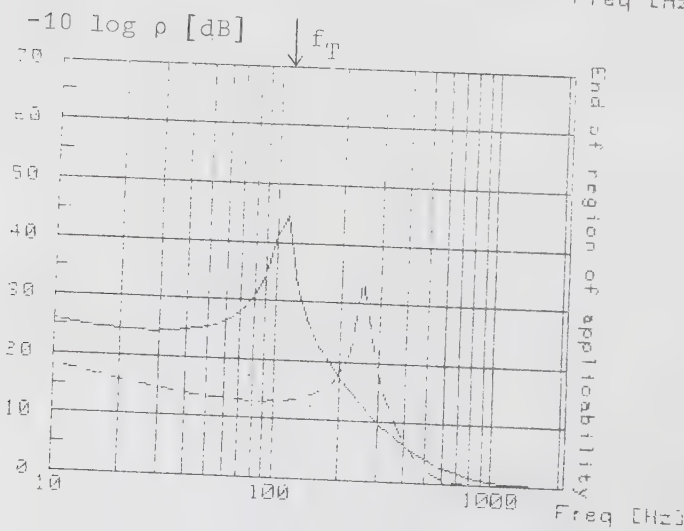
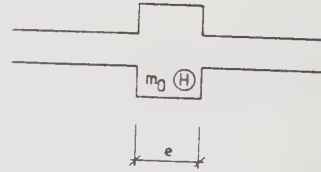
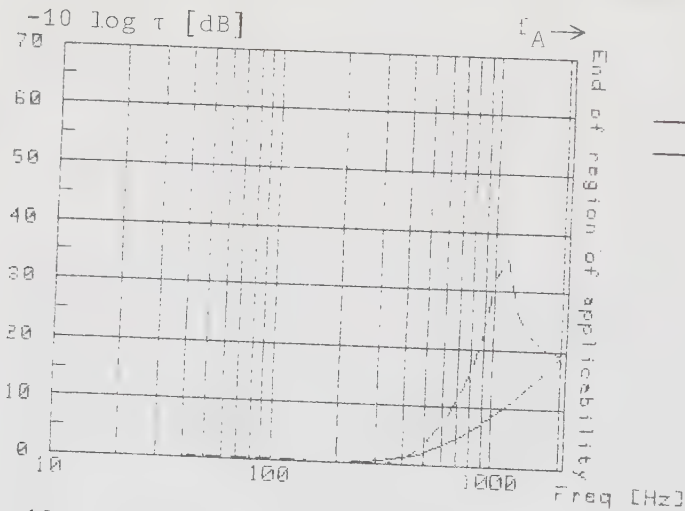
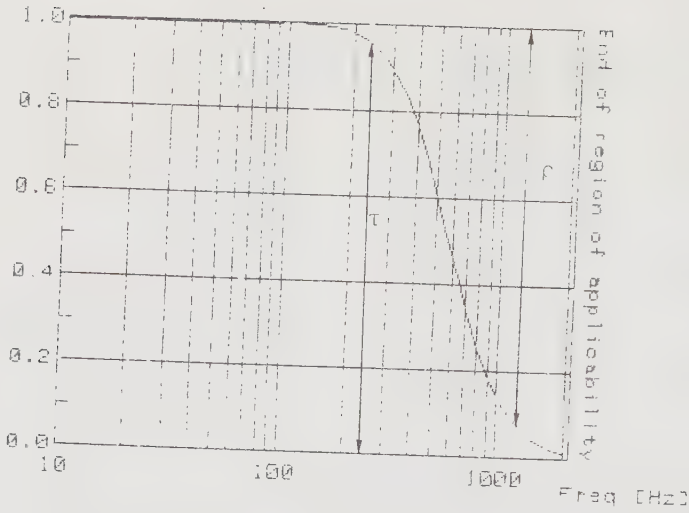
or

$$e > \frac{m_0}{2m} \quad (29)$$

In the practical example from 1.5.1 we have $e = 0.16$ m and $m_0/2m = 0.32$ m.

Between the frequencies f_T and f_A , the transmission loss will be somewhat lower when finite clamping length is considered, but above f_A it will be slightly higher.

Below we show the example from 1.5.1 (---→) compared to the result we get when the finite clamping length is considered (←-----→).



1.5.3 Excentric blocking mass on a beam

[The reciprocal problem, that of an incident longitudinal wave and a transmitted bending wave has been analyzed approximately by Müller (1957) [5], and in [2] (1967) Cremer derives the more exact expression for this "inverse" transmission efficiency τ_{LB} .]

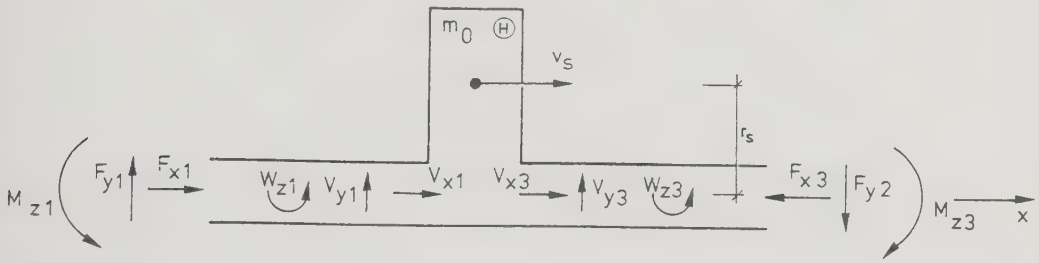


Fig 1.5.3. The excentric blocking mass is assumed to be rigidly connected to the beam. It has the mass m_0 , the center of gravity is at a distance r_s from the center line of the beam, and it has the mass moment of inertia θ_0 at the center of gravity.

For a rectangular cross section we have:

$$m_0 = \rho \cdot h_0 \cdot b_0 \quad [\text{kg/m}] \quad (30)$$

$$\theta_0 = \rho(b_0 h_0^3 + h_0 b_0^3)/12 \quad [\text{kg m}] \quad (31)$$

where $b_0 \times h_0$ are the dimensions of the blocking mass. (The z -dimension of the beam has been left out in the above equations.)

We now again disregard the finite clamping length and assume the mass to be point-attached to the beam.

The boundary conditions here are similar to those in part 1.5.1, but attention must be paid to the fact that the bending waves cause rotational acceleration

of the mass, which leads to axial reaction forces in the beam, which again produces secondary longitudinal waves there.

The boundary conditions ii), iii) and iv) from 1.5.1 can be assumed to be valid here too, but the moment condition i) needs some adjustment. We do this by dividing the rotational movement of the mass around the center line of the beam into an axial displacement of the center of gravity and a rotation around this:

$$\begin{aligned} \text{i)} \quad M_{Z_1} - M_{Z_3} &= j\omega\theta_0 w_{Z_2} - j\omega m_0 r_S v_S \\ \text{ii)} \quad F_{Y_1} - F_{Y_3} &= j\omega m_0 v_{Y_3} \\ \text{iii)} \quad v_{Y_1} &= v_{Y_3} \quad (= v_{Y_2}) \\ \text{iv)} \quad w_{Z_1} &= w_{Z_3} \end{aligned}$$

We now need two extra boundary conditions for the longitudinal wave generated in the beam:

$$\begin{aligned} \text{v)} \quad v_{X_1} &= v_{X_3} \\ \text{vi)} \quad F_{X_1} - F_{X_3} &= j\omega m_0 v_S \end{aligned}$$

where v_S is the velocity of the center of gravity:

$$v_S = v_{X_3} - r_S w_{Z_2} \quad (32)$$

We now combine the conditions v) and vi) with the help of the longitudinal wave impedance:

$$F_{X_1} = -Z_L v_{X_1} \quad (33)$$

$$F_{X_3} = Z_L v_{X_3} \quad (34)$$

The result is

$$F_{X_1} - F_{X_3} = j\omega m_0 v_S = -2 Z_L v_{X_3} \quad (35)$$

Now we combine (32) and (35) and eliminate v_{X_3} which gives us:

$$v_S \left(1 + \frac{j\omega m_0}{2 Z_L}\right) = -r_S w_{Z_2} \quad (36)$$

After substitution of (36) into boundary condition i) it becomes:

$$i) M_{Z_1} - M_{Z_3} = j\omega w_{Z_2} \left(\Theta_0 + \frac{m_0 r_S^2}{1 + \frac{j\omega m_0}{2 Z_L}} \right)$$

We then finally assume $w_{Z_2} = w_{Z_3}$ and now all the boundary conditions can be described with help of the parameters r , r_j , t and t_j as before.

If we take a closer look at boundary condition i), we see that if $Z_L \gg \omega m_0$ (low frequency) we have the same boundary condition as for the symmetric blocking mass. $\Theta_0 + m_0 r_S^2$ is the mass moment of inertia for the blocking mass taken at the central axis of the beam.

In fact if we set $\Theta = \Theta_0 + m_0 r_S^2$, boundary condition i) becomes:

$$i) M_{Z_1} - M_{Z_3} = j\omega w_{Z_3} \left(\Theta - \frac{m_0 r_S^2}{1 - j\frac{\omega m_0}{2 Z_L}} \right)$$

At high frequencies, where $Z_L \ll \omega m_0$, we have again the same boundary condition, but now with the mass moment of inertia Θ_0 .

If we now combine the boundary conditions in an equation system as before, it is convenient to introduce a new help-variable:

$$\gamma = \frac{m_0 r_S^2}{2 Z_L} \left(\frac{\omega^2}{Bk_B} \right) \quad (37)$$

With the help of α and δ defined as in the same way as in part 1.5.1 and the γ above we have:

$$\begin{bmatrix} -1 & +1 & (1+j(\alpha-\gamma)) & (-1+(\alpha-\gamma)) \\ -j & +1 & (-j+\delta) & (1+\delta) \\ +1 & +1 & -1 & -1 \\ +j & +1 & +j & +1 \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -j \\ -1 \\ +j \end{bmatrix} \quad (38)$$

This gives the following expressions:

$$r = \frac{-4\delta + 4(\alpha - \gamma) + 2(\alpha - \gamma)\delta}{(4\delta + 4(\alpha - \gamma)) + j(-16 - 4\delta + 4(\alpha - \gamma) + 2(\alpha - \gamma)\delta)} \quad (39)$$

$$r_j = \frac{-4(\alpha - \gamma) - 2(\alpha - \gamma)\delta + j(4\delta - 2(\alpha - \gamma)\delta)}{(4\delta + 4(\alpha - \gamma)) + j(-16 - 4\delta + 4(\alpha - \gamma) + 2(\alpha - \gamma)\delta)} \quad (40)$$

$$t = \frac{-j(16 + 4\delta - 4(\alpha - \gamma))}{(4\delta + 4(\alpha - \gamma)) + j(-16 - 4\delta + 4(\alpha - \gamma) + 2(\alpha - \gamma)\delta)} \quad (41)$$

$$t_j = \frac{4(\alpha - \gamma) + j4\delta}{(4\delta + 4(\alpha - \gamma)) + j(-16 - 4\delta + 4(\alpha - \gamma) + 2(\alpha - \gamma)\delta)} \quad (42)$$

As we have identical beams on each side of the blocking mass we obtain the transmission and reflection efficiencies for bending waves:

$$\tau = \tau_{BB} = |t|^2 \quad (43)$$

$$\rho = \rho_{BB} = |r|^2 \quad (44)$$

but now $\tau_{BB} + \rho_{BB} \neq 1$ as we also have transmitted and reflected longitudinal waves:

$$\tau_{BB} + \rho_{BB} + \tau_{BL} + \rho_{BL} = 1 \quad (45)$$

We derive τ_{BL} as:

$$\tau_{BL} = \frac{c_L}{2 c_B} \left| \frac{v_{x3}}{v_{y1+}} \right|^2 \quad (46)$$

$$\tau_{BL} = \frac{c_L c_B}{2\omega^2 r_S^2} \left| \frac{\gamma}{\delta} \right|^2 |jt + t_j|^2 \quad (47)$$

As one of the boundary conditions was $v_{x1} = v_{x3}$ we also have:

$$\rho_{BL} = \frac{c_L}{2 c_B} \left| \frac{v_{x1}}{v_{y1+}} \right|^2 = \tau_{BL} \quad (48)$$

It was mentioned at the beginning of this part that Cremer had derived an expression for τ_{LB} in [2]. For reasons of reciprocity we should have $\tau_{BL} = \tau_{LB}$, and after some manipulation with equation (46) we arrive at the same expression as Cremer's τ_{LB} .

At low frequencies we would expect only small deviations from the results obtained for a symmetric mass with the same θ and m_0 .

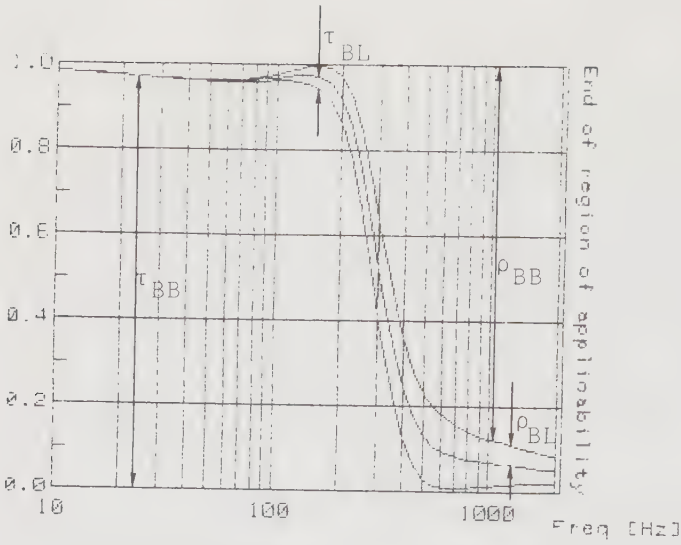
At high frequencies we also expect similar results to those obtained for a symmetric mass with the same m_0 but the somewhat lower mass moment of inertia θ_0 .

At intermediate frequencies we would expect some changes in ρ_{BB} and τ_{BB} as a part of the incident bending wave power is used in generating the longitudinal waves.

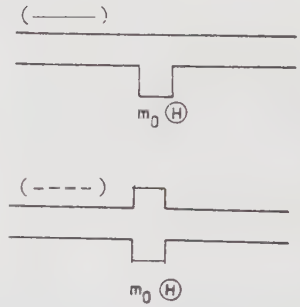
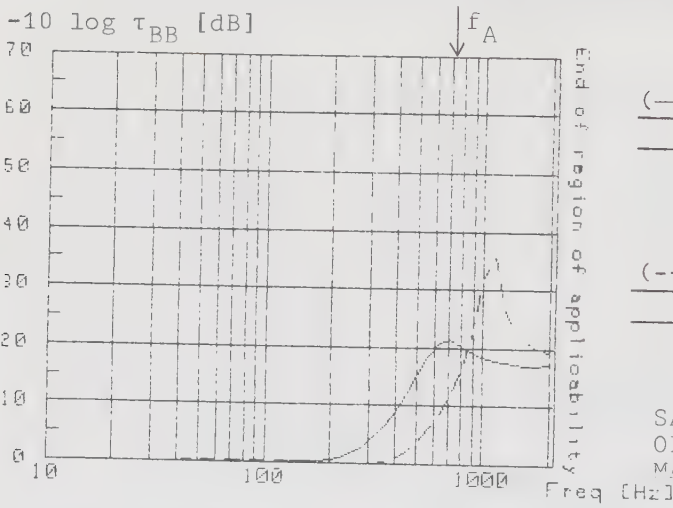
Below we show the results for the same beam as in previous examples provided with an excentric blocking mass ($b_0 \times h_0 = 2h \times 4h$). For comparison we show the curves (----) for the same blocking mass placed symmetrically ($b_0 \times h_0 = 2h \times 5h$).

MATERIALS AND DIMENSIONS

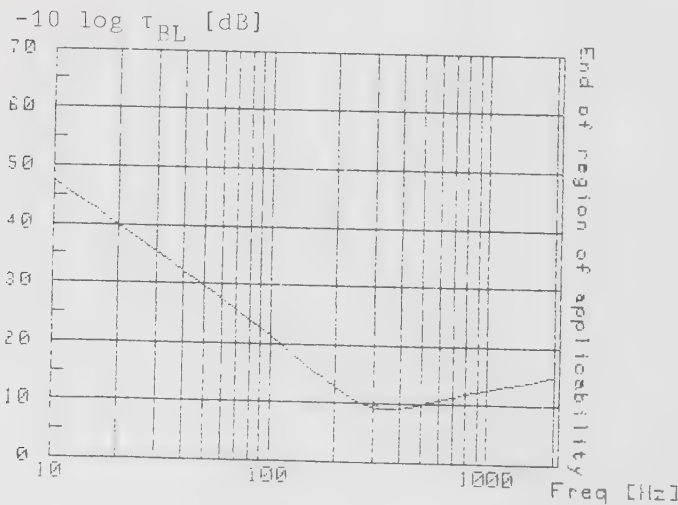
ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10		2.6E+10
Density	Rho[kg/m ³]	2300		2300
Poiss. ratio	My [1]	0.25		0.25
Thickness	H [m]	0.080		0.080
Excentric Blocking				
Mass	Bo x Ho [m]		0.160 x 0.320	
Extra Mass	Mo [kg/m]		118	
MassMom of Inert	[kgm]		4.3E+00	

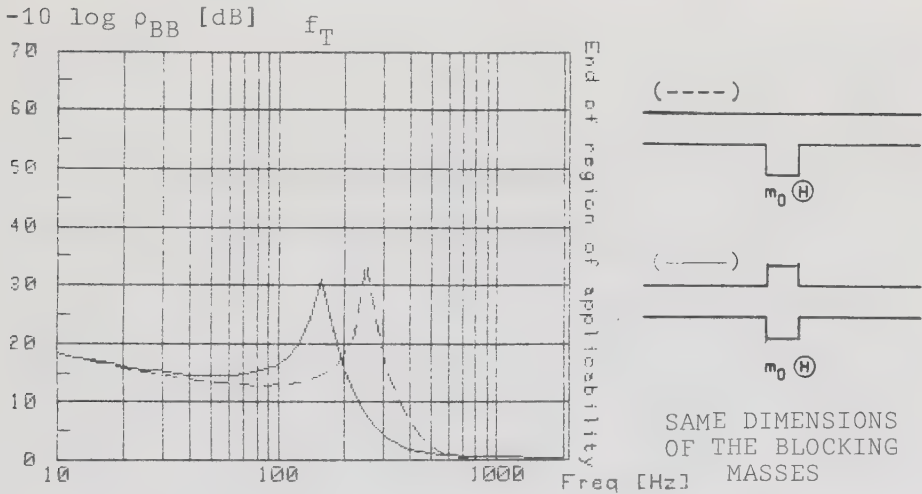
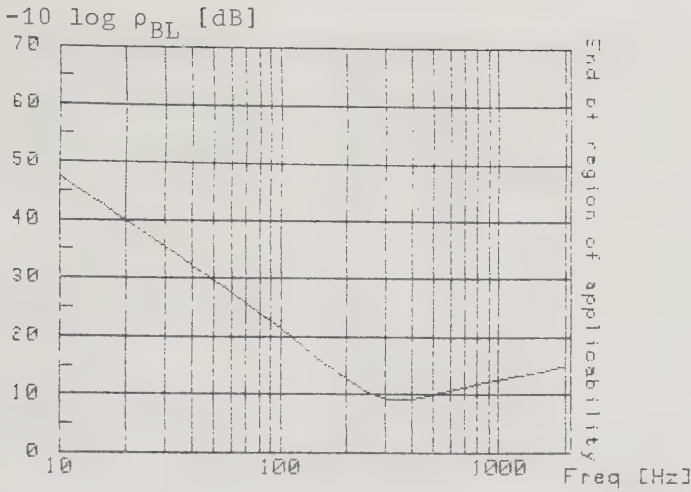


Transmission and Reflection Efficiencies for a Bending Wave at an Excentric Blocking Mass [Bt]

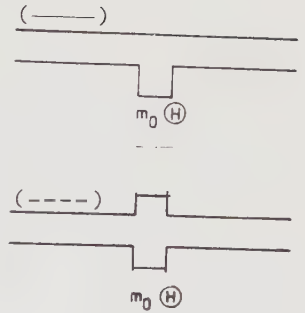
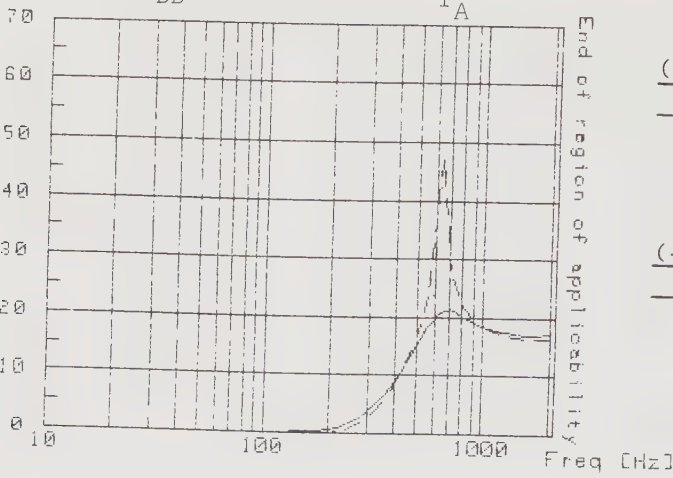


SAME DIMENSIONS OF THE BLOCKING MASSES

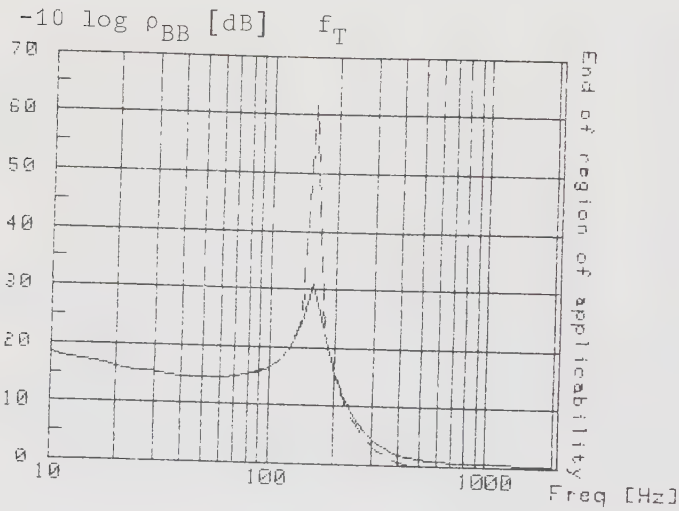




We have larger θ for the excentric case and thus lower f_A and f_T . However if we for comparison show the curves for a symmetric blocking mass instead, which has the same value of m_0 and θ as the excentric one, we get the following result:

$-10 \log \tau_{BB} \text{ [dB]}$ 

SAME VALUES FOR
 m_0 AND θ FOR
 BOTH CASES

 $-10 \log \rho_{BB} \text{ [dB]}$ 

Here we have the same values for f_A and f_T in both cases, and only a marginal difference in the curves, except near the maximum values at f_A and f_T respectively.

1.6 Practical boundary conditions for beams separated with an elastic interlayer

1.6.1 Extra mass on one side

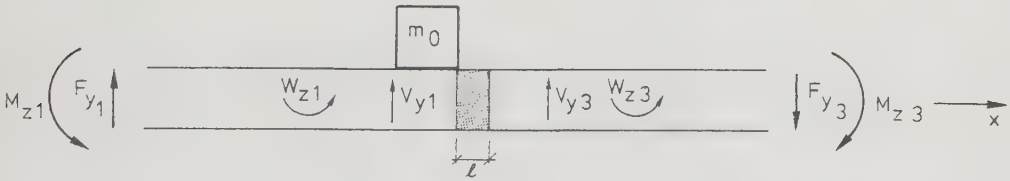


Fig 1.6.1. The mass is assumed to have a hinged connection to the beam, so that it follows its lateral movements, but does not take up any moments. ($\theta = 0$).

The boundary conditions are the same as the ones without the extra mass (chapter 1.3.1), except for the shear forces:

- i) $M_{z1} = M_{z3}$
- ii) $F_{y1} - m_0 j \omega v_{y1} = F_{y3}$
- iii) $(v_{y1} - v_{y3}) / j \omega = F_{y3} / K$
- iv) $(w_{z1} - w_{z3}) / j \omega = M_{z3} / C$

The equation system for this becomes with the same help-variables as before (v , β , δ):

$$\begin{bmatrix} -1 & +1 & +1 & -1 \\ (\delta - j) & (1 + \delta) & -j & +1 \\ +1 & +1 & -(1 + j\beta) & (-1 + \beta) \\ +j & +1 & (-v + j) & (1 + v) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -(\delta + j) \\ -1 \\ +j \end{bmatrix}$$

Here the expressions for r and t are:

$$r = \frac{(-4\nu+4\beta+2\nu\beta) + \delta(-4-4\nu+2\beta+2\nu\beta) + j\delta(-4\beta-2\nu\beta)}{\{(4\nu+4\beta) + \delta(4+4\nu-2\nu\beta)\} + j\{(-16-4\nu+4\beta+2\nu\beta) + \delta(-4+6\beta+2\nu\beta)\}}$$

(1)

$$t = \frac{j\{(-16-4\nu+4\beta) + \delta(-4+4\beta)\}}{\{(4\nu+4\beta) + \delta(4+4\nu-2\nu\beta)\} + j\{(-16-4\nu+4\beta+2\nu\beta) + \delta(-4+6\beta+2\nu\beta)\}}$$

(2)

If the extra mass is placed on the other side we get the same expression, except that the imaginary part in the numerator of r changes sign.

We see that for very small values of δ the terms including δ can be neglected (small mass, low frequency) and the expressions above become the same as those of chapter 1.3.1.

On the other hand if we have very large values of δ , the terms including δ become dominant, and we get the same expressions as for one end simply supported (chapter 1.4.3).

For δ -values somewhere between these extremes we would expect to find a minimum in r somewhere close to the frequency of total transmission f_T from chapter 1.3.1.

We see that we have a possibility for a frequency of total attenuation, when the numerator of t becomes zero:

$$4\beta(1 + \delta) = 4\nu + 16 + 4\delta \quad (3)$$

As β grows more quickly with the frequency than ν does, we get a successively lower frequency of total attenuation, f_A when the mass m_0 is increased.

Above f_A , when the terms including the highest powers of frequency start to dominate we get the asymptote:

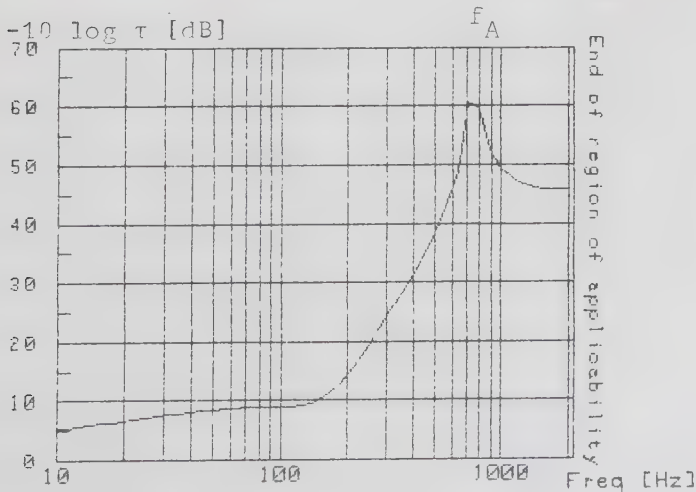
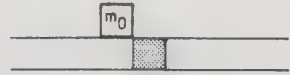
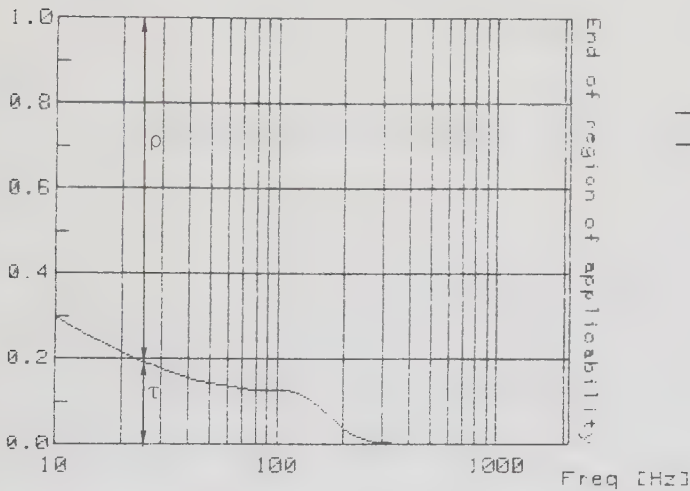
$$R = -10 \log \tau \approx 10 \log (\nu^2/2) \quad (4)$$

which increases as $10 \log f$ and is the same as we previously got for a simply supported end.

Below we show as an example the same case as before with the extra mass $m_0 = 200 \text{ kg/m}$.

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m ³]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	
Extra mass	Mo [kg/m]	200		



1.6.2 Equal extra masses on each side

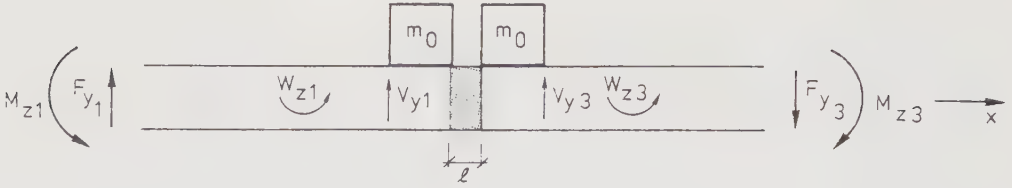


Figure 1.6.2. The masses are assumed to have a hinged connection to the beams, so that they follow the lateral movements, but do not take up any moments. ($\theta = 0$).

The boundary conditions are similar to those in part 1.6.1, except that the shear force corrections now appear on both sides:

- i) $M_{z1} = M_{z3}$
- ii) $F_{y1} - m_0 j \omega v_{y1} = F_{y3} + m_0 j \omega v_{y3}$
- iii) $(v_{y1} - v_{y3})/j \omega = (F_{y3} + j \omega m_0 v_{y3})/K$
- iv) $(w_{z1} - w_{z3})/j \omega = M_{z3}/C$

The equation system is:

$$\begin{bmatrix} -1 & +1 & +1 & -1 \\ (-j+\delta) & (1+\delta) & (-j+\delta) & (1+\delta) \\ +1 & +1 & -(1+\beta(j-\delta)) & (-1+\beta(1+\delta)) \\ +j & +1 & (j-v) & (1+v) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ (-j-\delta) \\ -1 \\ +j \end{bmatrix}$$

(5)

This provides the following equations for r and t , which now have become rather long and tedious, so we choose to write them as:

$$r = \frac{(-4v+4\beta+2v\beta) + \delta(-8-8v+4\beta+4v\beta) + \delta^2(4\beta+4v\beta)}{N_r + j N_i} \quad (6)$$

$$t = \frac{j(-16-4v+4\beta+\delta(-8+8\beta)+\delta^2 4\beta)}{N_r + j N_i} \quad (7)$$

where: $N_r = (4v+4\beta) + \delta(8+8v-4v\beta) + \delta^2(-4\beta-4v\beta)$

$N_i = (-16-4v+4\beta+2v\beta) + \delta(-8+12\beta+4v\beta) + \delta^2 4\beta$

Here we also find that in the limiting case of small δ we get once again the same expressions as in chapter 1.3.1.

The other limiting case, that of large δ , is seen to give the same expressions as for both ends simply supported (chapter 1.4.4).

For δ -values between the extremes, we see that we now again have a possibility of total transmission, when the numerator of r becomes zero. For small masses, this frequency, f_T , would be about the same as in chapter 1.3.1, but as m_0 increases the term without δ in the numerator has to be negative and this means that f_T is lowered.

We also have a frequency of total attenuation when the numerator of t vanishes. For small values of m_0 we get about the same frequency, f_A , as in chapter 1.3.1. When m_0 successively increases f_A is lowered, because the term without δ must be negative and β increases more quickly with the frequency than v does. However as $f_A > f_T$ we have higher values of δ at f_A than at f_T (for the same m_0), which means that f_A

is lowered more than f_T for a certain increase in m_0 .

Above f_A we get the high frequency asymptote:

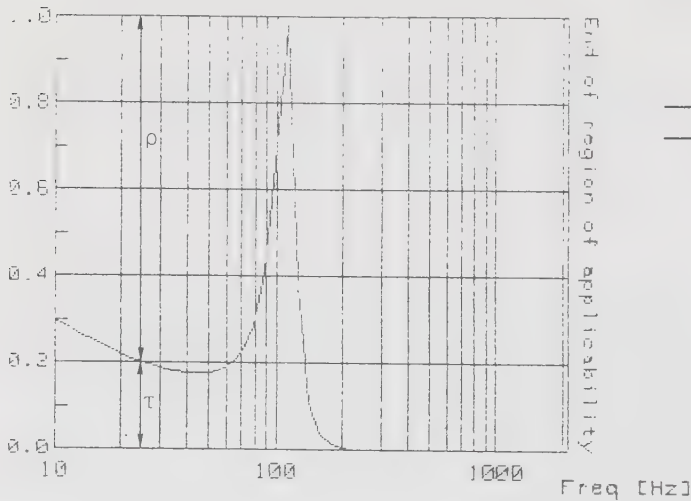
$$R = -10 \log \tau \approx 20 \log v \quad (8)$$

which again increases as $10 \log f$ and is the same as previously obtained for the case where both ends are simply supported.

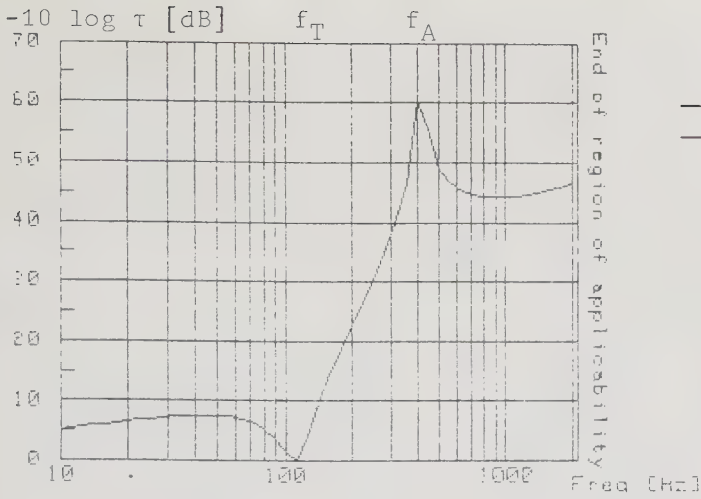
Below we show the same example as before with a mass, $m_0 = 100$ kg, on each side of the elastic layer.

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
Young modulus	E [Pa]	$2.6E+10$	$1.8E+07$	$2.6E+10$
Shear modulus	G [Pa]		$5.0E+06$	
Density	ρ [kg/m ³]	2300	800	2300
Poiss. ratio	ν	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
width of layer	L [m]		0.012	
Extra mass	M_0 [kg/m]	100		100



Transmission Efficiency for a Bending Wave
at a Junction with an Elastic Layer [Emm]



1.6.3 Different extra masses on each side

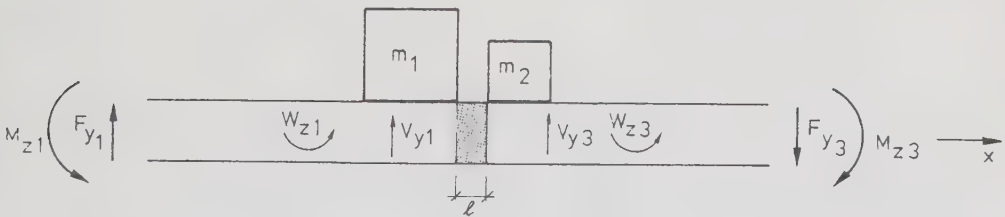


Fig 1.6.3. The masses are assumed to have a hinged connection to the beams, so that they follow the lateral movements but do not take up any moments. ($\theta_1 = 0$, $\theta_2 = 0$).

The boundary conditions are analogous to those in part 1.6.2:

- i) $M_{Z_1} = M_{Z_3}$
- ii) $F_{y_1} - m_{01} j \omega v_{y_1} = F_{y_3} + m_{03} j \omega v_{y_3}$
- iii) $(v_{y_1} - v_{y_3}) / j \omega = (F_{y_3} + m_{03} j \omega v_{y_3}) / K$
- iv) $(w_{Z_1} - w_{Z_3}) / j \omega = M_{Z_3} / C$

The equation system is:

$$\begin{bmatrix} -1 & +1 & +1 & -1 \\ (-j+\delta_1) & (1+\delta_1) & (-j+\delta_3) & (1+\delta_3) \\ +1 & +1 & -(1+\beta(j-\delta_3)) & (-1+\beta(1+\delta_3)) \\ +j & +1 & (j-\nu) & (1+\nu) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -j-\delta_1 \\ -1 \\ +j \end{bmatrix} \quad (9)$$

where δ_1 and δ_3 correspond to the masses m_{01} and m_{03} respectively.

Here the equations for r and t are also rather long and tedious, so we have chosen to write them as:

$$r = \frac{R_r + j R_i}{N_r + j N_i} \quad (10)$$

$$t = \frac{j T_i}{N_r + j N_i} \quad (11)$$

where $N_r = (4\nu+4\beta) + (\delta_1+\delta_3)(4+4\nu-2\nu\beta) + \delta_1\delta_3(-4\beta-4\nu\beta)$

$$N_i = (-16-4\nu+4\beta+2\nu\beta) + (\delta_1+\delta_3)(-4+6\beta+2\nu\beta) + \delta_1\delta_3(4\beta)$$

$$R_r = (-4\nu+4\beta+2\nu\beta) + (\delta_1+\delta_3)(-4-4\nu+2\beta+2\nu\beta) + \delta_1\delta_3(4\beta+4\nu\beta)$$

$$R_i = (\delta_1-\delta_3)(-4\beta-2\nu\beta)$$

$$T_j = (-16-4\nu+4\beta) + (\delta_1+\delta_3)(-4+4\beta) + \delta_1\delta_3 \cdot 4\beta$$

We see that if $\delta_3 = 0$ we get the same expressions as in part 1.6.1 and if $\delta_1 = \delta_3 = \delta$ we get the same expressions as in part 1.6.2.

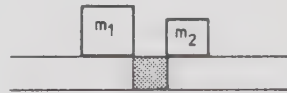
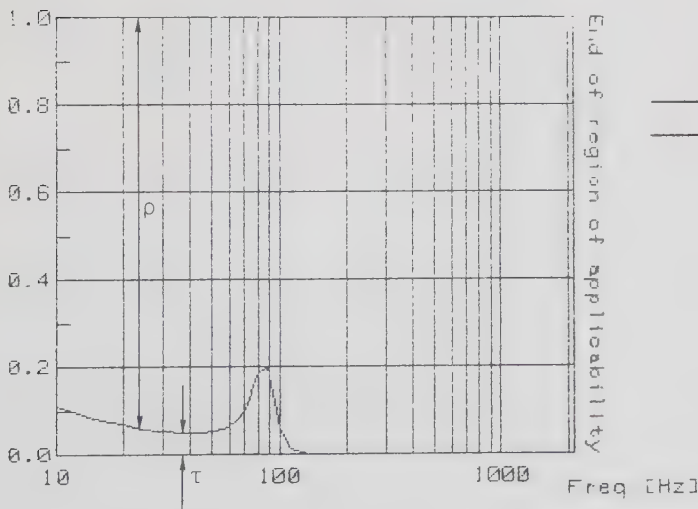
The resulting curve for the transmission loss should have the same tendencies as explained in parts 1.6.1 and 1.6.2.

If the masses differ a lot, then the curve lies close to the one in part 1.6.1 and if they are similar the curve would lie close to the one in part 1.6.2.

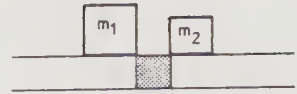
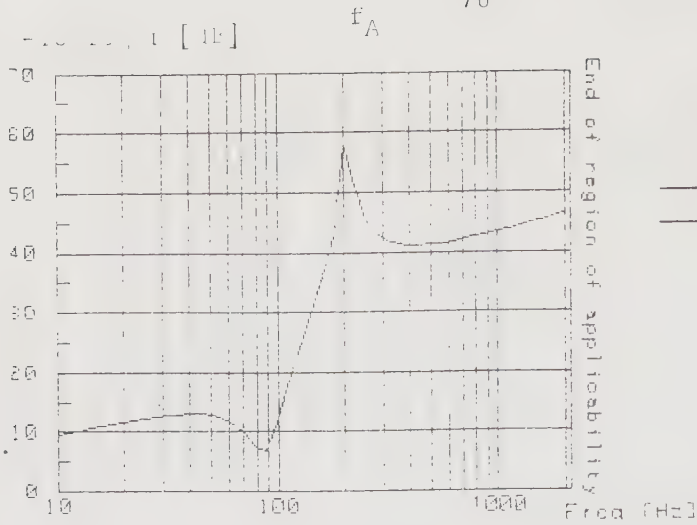
Below we show once again the same example, here with masses $m_{01} = 100 \text{ kg/m}$ and $m_{03} = 500 \text{ kg/m}$. This rather large mass should give a low frequency of total attenuation, f_A .

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m ³]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	
Extra mass	Mo [kg/m]	100		500



Transmission Efficiency for a Bending Wave
at a Junction with an Elastic Layer [Em12]



1.6.4 Symmetric blocking mass combined with an elastic layer

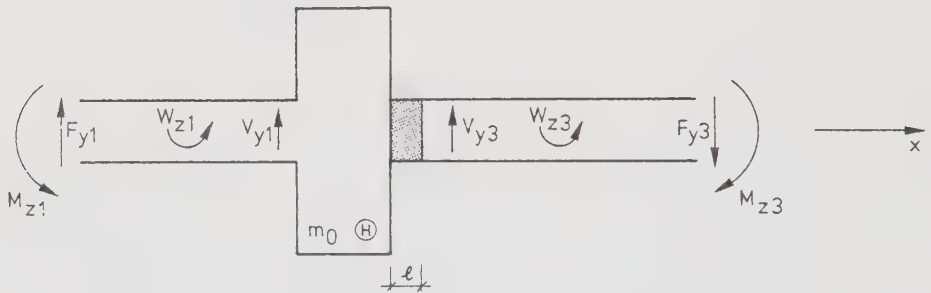


Fig 1.6.4. The blocking mass is assumed to be rigidly connected to the beam.

The boundary conditions from part 1.6.1 are valid here, except the one for the moments, which has to be complemented with the mass moment of inertia for the blocking mass:

- i) $M_{Z_1} - M_{Z_3} = j\omega\Theta w_{Z_1}$
 ii) $F_{y_1} - F_{y_3} = j\omega m_0 v_{y_1}$
 iii) $(v_{y_1} - v_{y_3})/j\omega = F_{y_3}/K$
 iv) $(w_{Z_1} - w_{Z_3})/j\omega = M_{Z_3}/C$

Using the same help variables as before the equation system becomes:

$$\begin{bmatrix} -(1+j\alpha) & (1-\alpha) & +1 & -1 \\ (\delta-j) & (1+\delta) & -j & +1 \\ +1 & +1 & -(1+j\beta) & (-1+\beta) \\ +j & +1 & (j-v) & (1+v) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} (1-j\alpha) \\ -(j+\delta) \\ -1 \\ +j \end{bmatrix} \quad (12)$$

This gives the equations for r and t , written in the same way as in part 1.6.3:

$$r = \frac{R_r + j R_i}{N_r + j N_i} \quad (13)$$

$$t = \frac{j T_i}{N_r + j N_r} \quad (14)$$

$$\text{where: } N_r = (4v+4\beta) + \delta(4+4v-2v\beta) + \alpha\{(4-4\beta-2v\beta) - \delta(2\beta+2v)\}$$

$$N_i = (-16-4v+4\beta+2v\beta) + \delta(-4+6\beta+2v\beta) + \alpha\{(4+6v-2v\beta) + \delta(2+2v-2\beta-2v\beta)\}$$

$$R_r = (-4v+4\beta+2v\beta) + \delta(-4-4v+2\beta+2v\beta) + \alpha\{(4+2v-4\beta-2v\beta) + \delta(2+2v-2\beta-2v\beta)\}$$

$$R_i = \delta(-4\beta-2v\beta) + \alpha\{(4v-2v\beta) + \delta(2\beta+2v)\}$$

$$T_i = (-16-4v+4\beta) + \delta(-4+4\beta) + \alpha(4+4v)$$

If the blocking mass is placed on the other side of the elastic layer instead, the only change in the above expressions is that R_i changes sign.

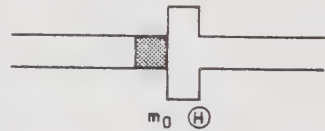
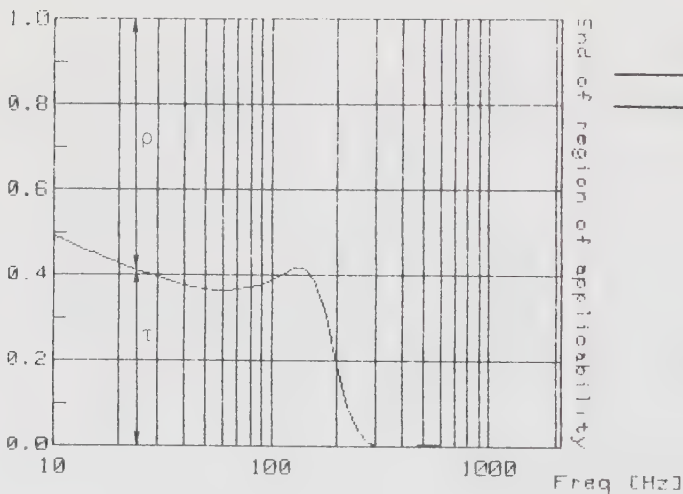
We see that $\alpha \rightarrow 0$ gives the expressions derived in part 1.6.1 and $\alpha \rightarrow \infty$ gives the expressions for a guided end in part 1.4.1 if we set $\delta = 0$.

As a practical example we use the same beam and elastic layer as before. A realistic blocking mass would be an excentric one with ($b_0 \times h_0 = 2h \times 2h$).

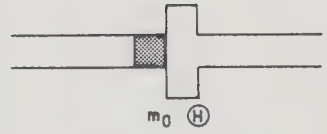
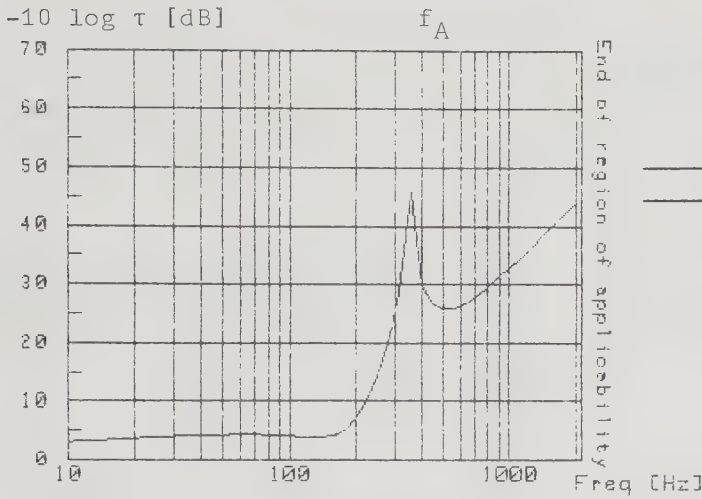
In the following example we use a symmetric blocking mass with the same θ and m_0 and the same height as this excentric blocking mass has. (m_0 is calculated separately and used as an input parameter.)

MATERIALS AND DIMENSIONS

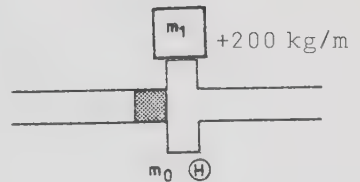
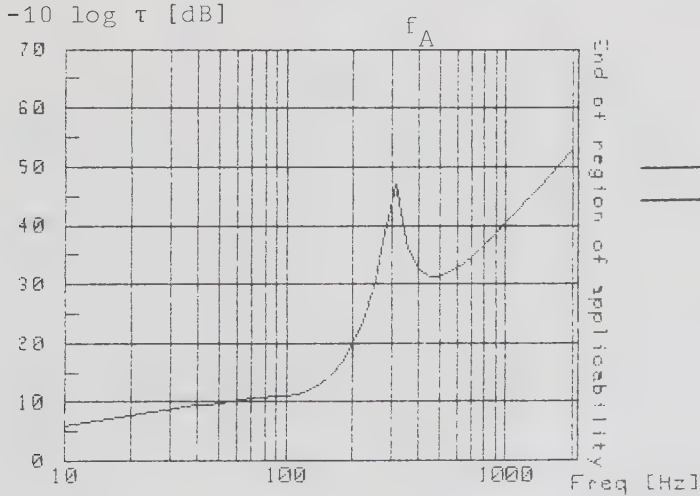
ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m3]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	
Symmetric Blocking				
Mass	Bo x Ho [m]	0.224x0.240		
Extra Mass	Mo [kg/m]	59		
MassMom of Inert	[kgm]	1.1E+00		



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As a further example we place an extra mass of 200 kg/m on the blocking mass above (load from the lightweight wall) and get the following curve for $-10 \log \tau$:



This lowers the frequency f_A , and also increases the transmission loss both below and above it.

1.6.5 Symmetric blocking masses on each side of an elastic layer

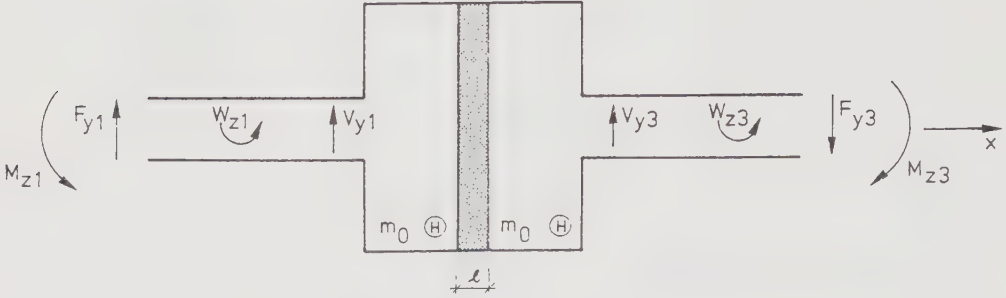


Fig 1.6.5. The blocking masses are assumed to be rigidly connected to the beams and they each have the mass m_0 and the mass moment of inertia Θ .

We can use the same boundary conditions as in part 1.6.2 with the moment condition complemented with the mass moments of inertia for the blocking masses:

- i) $M_{z1} - j\omega \Theta w_{z1} = M_{z3} + j\omega \Theta w_{z3}$
- ii) $F_{y1} - j\omega m_0 v_{y1} = F_{y3} + j\omega m_0 v_{y3}$
- iii) $(v_{y1} - v_{y3})/j\omega = (F_{y3} + j\omega m_0 v_{y3})/K'$
- iv) $(w_{z1} - w_{z3})/j\omega = (M_{z3} + j\omega \Theta w_{z3})/C'$

Here the spring constants K and C have been replaced by K' and C' as the contact height for the elastic layer is not h as before but h_0 . In accordance to that we use the help variables

$$\beta = \frac{\omega c_B m}{K'} \quad (15)$$

$$\nu = \frac{B k_B}{C'} \quad (16)$$

together with the same α and δ as before, which gives us the following equation system:

$$\begin{bmatrix} (-1-j\alpha) & (1-\alpha) & (1+j\alpha) & (-1+\alpha) \\ (-j+\delta) & (1+\delta) & (-j+\delta) & (1+\delta) \\ +1 & +1 & -(1+\beta(j-\delta)) & (-1+\beta(1+\delta)) \\ +j & +1 & (j-v(1+j\alpha)) & (1+v(1-\alpha)) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} (1-j\alpha) \\ (-j-\delta) \\ -1 \\ +j \end{bmatrix} \quad (17)$$

Solving the above equation to obtain r and t now leads to:

$$r = \frac{R_r}{N_r + j N_i} \quad (18)$$

$$t = \frac{j T_i}{N_r + j N_i} \quad (19)$$

$$\begin{aligned} \text{where: } N_r &= (4v+4\beta)+\delta(8+8v-4v\beta) + \delta^2(-4\beta-4v\beta) \\ &\quad +\alpha\{(8-8\beta-4v\beta) + \delta(-8v-8\beta) + \delta^2(4v\beta)\} \\ &\quad +\alpha^2\{(-4v+4v\beta) + \delta(4v\beta)\} \\ N_i &= (-16-4v+4\beta+2v\beta) + \delta(-8+12\beta+4v\beta) + \delta^2 4\beta \\ &\quad +\alpha\{(8+12v-4v\beta) + \delta(8+8v-8\beta-12v\beta) + \delta^2(-4\beta-4v\beta)\} \\ &\quad +\alpha^2\{-4v+\delta(4v\beta-4v) + \delta^2(2v\beta)\} \\ R_r &= (-4v+4\beta+2v\beta) + \delta(-8-8v+4\beta+4v\beta) + \delta^2(4\beta+4v\beta) \\ &\quad +\alpha\{(8+4v-8\beta-4v\beta) + \delta(8+8v-8\beta-4v\beta) + \delta^2(-4\beta-4v\beta)\} \\ &\quad +\alpha^2\{(-4v+4v\beta) + \delta(-4v+4v\beta) + \delta^2 2v\beta\} \\ T_i &= (-16-4v+4\beta) + \delta(-8+8\beta) + \delta^2 4\beta \\ &\quad +\alpha\{8+8v\} - \alpha^2 4v \end{aligned}$$

The limiting case of $\alpha \rightarrow 0$ gives us the same expressions as derived in part 1.6.2 and we also see that when $\alpha \rightarrow \infty$ and $\delta \rightarrow 0$ we have the same expressions as we had for the case where both ends were guided.

The expressions above are rather complicated and difficult to interpret. We can however start with the fact that β and especially v according to (15) and (16) have much smaller values than before. If we consider the influence of this on the original case ($\alpha \rightarrow 0$ and $\delta \rightarrow 0$ in the expression above), total transmission occurs when

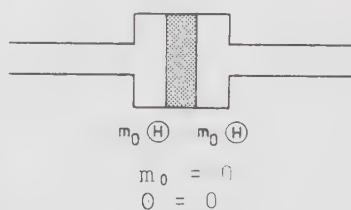
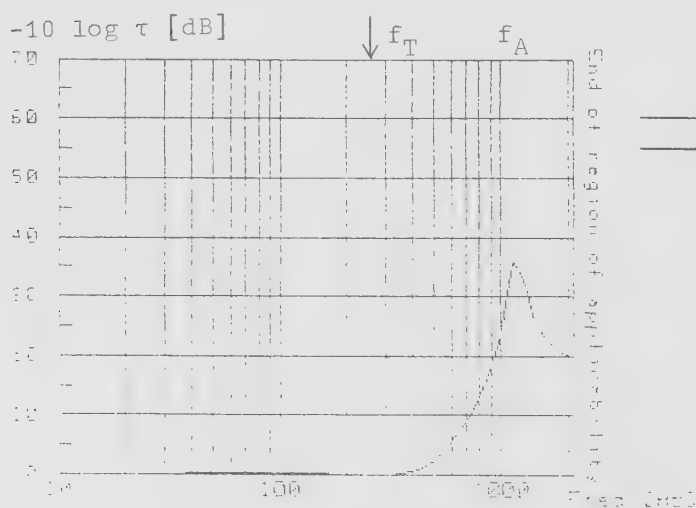
$$4\beta + 2v\beta = 4v \quad (20)$$

which may be at a frequency somewhat different from that of the original case. If h_2 is only slightly larger than h we may still use the approximate solution $\beta \approx 2$, and f_T will be slightly higher than in the original case. However if h_2 is made much larger than h , the above approximate solution is no longer valid. According to (15) and (16) v decreases more than β and f_T would be reduced instead.

Similarly in the equation for total attenuation, we have, as long as the approximate equation $\beta \approx v$ is valid, that f_A gets lower for larger h_2 . However, when h_2 gets too large, we have another approximate equation: $\beta \approx 4$ and this means higher f_A for larger h_2 .

For practical values ($2h < h_2 < 5h$) the main result of an increased h_2 would be to lower f_A down to the frequency region of interest. This would give us a similar type of curve for the transmission loss as for a blocking mass.

Below is the curve for $h_2 = 3h$ ($\alpha = 0$, $\delta = 0$).



Above f_A we have the asymptotic value

$$R = -10 \log \tau = 10 \log \left(1 + \left(\frac{v}{2}\right)^2\right) \quad (21)$$

which increases as $10 \log f$.

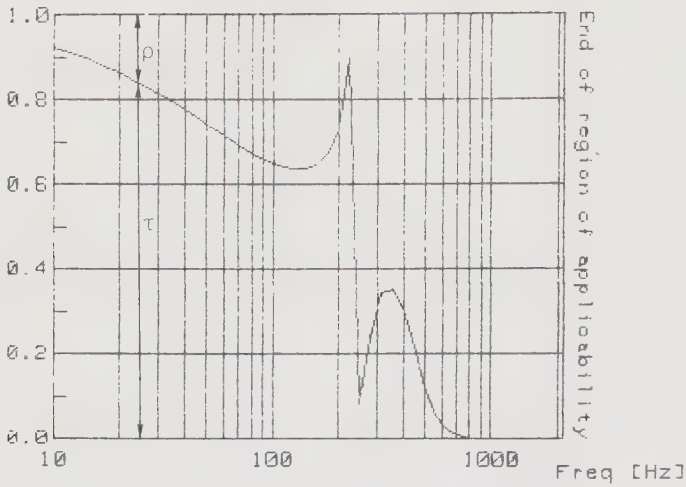
We see that as v decreases quickly with increasing h_2 , our transmission loss curve becomes more and more like a single top at f_A when h_2 increases successively.

However, we do have a mass and a mass moment of inertia on both sides of the elastic layer. As we discussed in part 1.6.2, the influence of the extra mass (the δ -parameter) is mainly to lower f_A , and to a certain extent f_T too. This is a phenomenon we would expect here too, but we must also consider the influence of the mass moment of inertia (the α -parameter). As shown in part 1.5.1, this too leads to frequencies of total attenuation and total transmission. Let us denote them as $f_{T\alpha}$ and $f_{A\alpha}$.

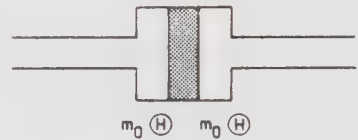
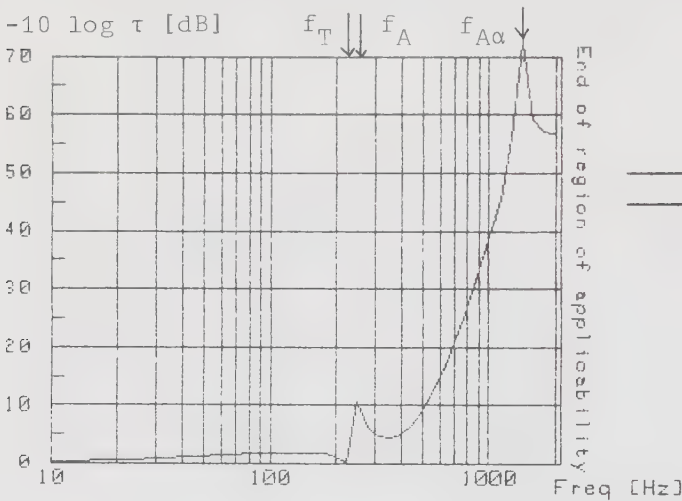
Below we show as a practical example the same symmetric mass as the one used in 1.6.4 so as to resemble the excentric mass ($b_0 \times h_0 = 2h \times 2h$), but here placed on both sides of the elastic layer.

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m3]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.240	0.080
Width of layer	L [m]		0.012	
Symmetric Blocking				
Masses	Bo x Ho [m]	0.224x0.240		0.224x0.240
Extra Mass	Mo [kg/m]	59		59
MassMom of Inert	[kgm]	1.1E+00		1.1E+00



Transmission Efficiency for a Bending Wave
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and Symmetric Blocking Masses on both Ends [EBxx]



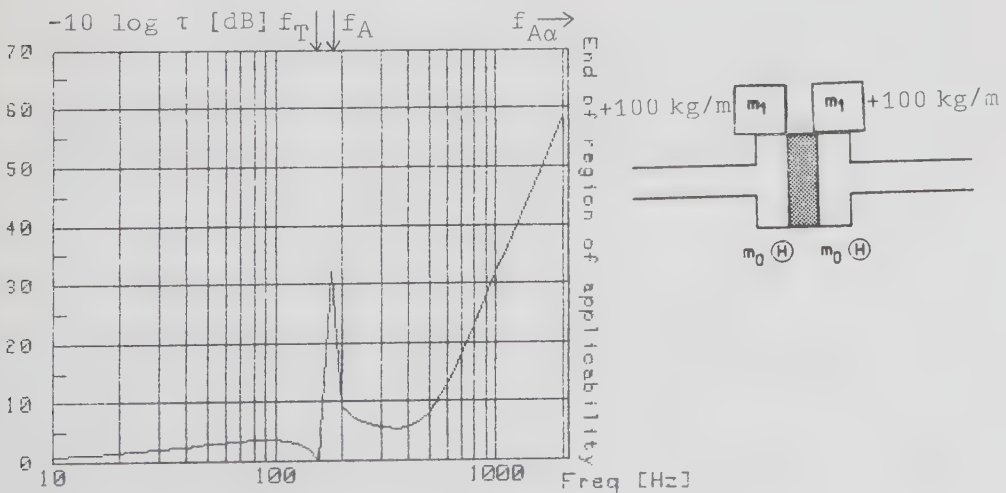
As we predicted, f_T and especially f_A are lowered, which gives a little improvement in the transmission loss. However this improvement is much smaller than we might have expected according to the results of part 1.6.2. First we must remember that ν and β have lower values here than in our original examples, which means a

smaller transmission loss above f_A .

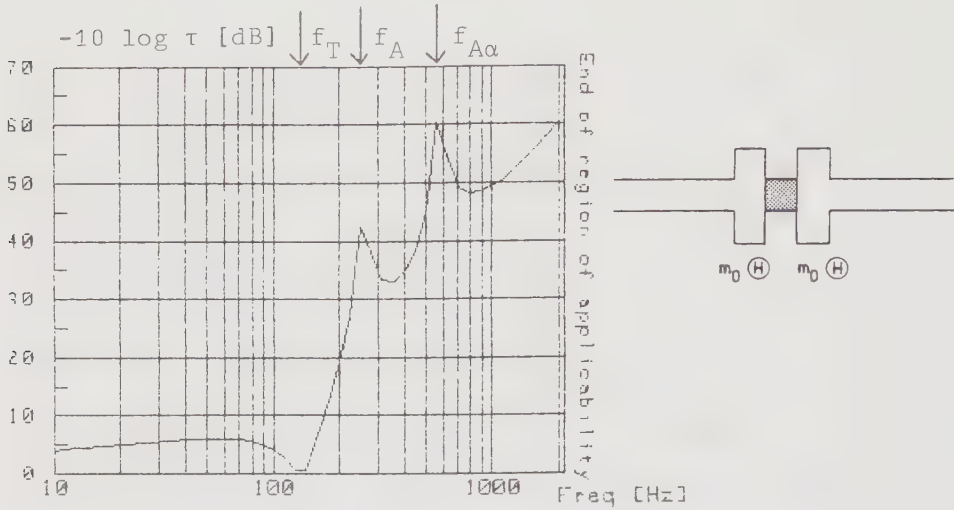
Secondly we have the influence of the mass moment of inertia. We may point out that in the example in part 1.5.1 we had m_0 and θ_0 that corresponded to the sum of our present parameters on each side of the elastic layer. These values gave $f_{T\alpha}$ at about the same frequency as our present f_A , and $f_{A\alpha}$ at about 1100 Hz. The improvement we get by a lower f_A is "eaten up" by the "total transmission" phenomenon for the mass moment of inertia at $f_{T\alpha}$. We would get higher values for the transmission loss in this frequency region, if the blocking mass was hinged ($\alpha = 0$). However, at higher frequencies (near $f_{A\alpha}$) we get an improvement because of the mass moment of inertia.

If we try to improve the transmission loss by adding extra masses on both sides (no extra mass moment of inertia, just added mass) we only get a marginal effect.

The figure below shows the transmission loss, with an added mass of 100 kg/m on each side. (The blocking masses themselves are the same as before.)



Finally we show the result for the two blocking masses above, when the height of the elastic layer is limited to $h_2 = h$.



This gives us higher values for ν and β , a higher frequency f_A and an improved transmission loss above f_A . Again these values are deteriorated by the phenomenon at $f_{T\alpha}$, but less than before. The frequency $f_{A\alpha}$ is also lowered, and the total effect is a considerable improvement of the transmission loss in the frequency region above about 150 Hz. [Note that the frequency $f_{A\alpha}$ depends on more parameters than just α , although this notation has been chosen to distinguish it from the other attenuation frequency.]

1.6.6 An excentric blocking mass combined with an elastic layer

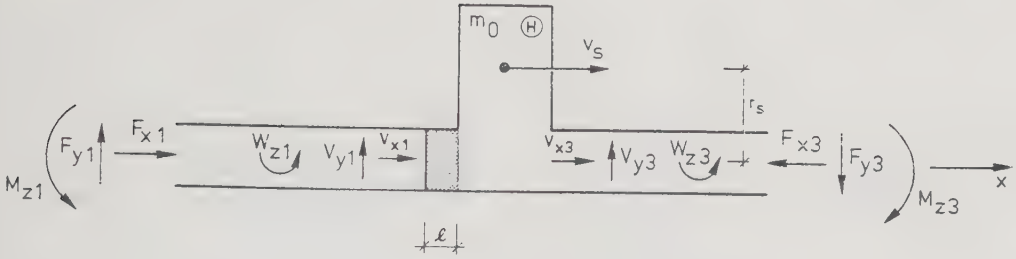


Fig 1.6.1. The excentric blocking mass is assumed to be rigidly connected to the beam. It has the mass m_0 , the center of gravity is at a distance r_s from the center line of the beam, and it has the mass moment of inertia θ_0 at the center of gravity.

The usual four boundary conditions are here:

- i) $M_{z1} - M_{z3} = j\omega\theta w_{z3} - j\omega m_0 r_s v_{x3}$
- ii) $F_{y1} - F_{y3} = j\omega m_0 v_{y3}$
- iii) $v_{y1} - v_{y3} = (j\omega/K) F_{y1}$
- iv) $w_{z1} - w_{z3} = (j\omega/C) M_{z1}$

Note that i) is exactly the same as in part 1.5.3, just written in a slightly different way. Similar to part 1.5.3 we need two extra boundary conditions for the longitudinal waves generated in the beams

- v) $v_{x1} - v_{x3} = (j\omega/K_x) F_{x1}$
- vi) $F_{x1} - F_{x3} = j\omega m_0 v_s$

The parameter K_x is defined in analogy with K (see equation (1.3.8)):

$$K_x = E_2 h_2 / l \quad (22)$$

In analogy with the calculations in part 1.5.3 we have:

$$v_S = v_{x_3} - r_S w_{z_3} \quad (23)$$

$$F_{x_1} = -Z_L v_{x_1} \quad (24)$$

$$F_{x_3} = Z_L v_{x_3} \quad (25)$$

We now combine conditions v) and vi) with the help of equations (23), (24) and (25), which gives us:

$$v_{x_3} = \frac{j\omega m_0 r_S w_{z_3}}{Z_L + j\omega m_0 + (1/Z_L + j\omega/K_x)^{-1}} \quad (26)$$

After substitution of (26) into the boundary condition i) we have:

$$i) \quad M_{z_1} - M_{z_3} = j\omega w_{z_3} \left(\theta - \frac{m_0 r_S^2}{1 - j \frac{Z_L}{\omega m_0} \left(\frac{2K_x + j\omega Z_L}{K_x + j\omega Z_L} \right)} \right)$$

It becomes convenient, once again, to introduce a new help parameter, similar to γ in part 1.5.3:

$$\gamma_x = \left(\frac{\omega^2}{Bk_B} \right) \frac{m_0 r_S^2}{1 - j \frac{Z_L}{\omega m_0} \left(\frac{2K_x + j\omega Z_L}{K_x + j\omega Z_L} \right)} \quad (27)$$

We may point out that if the interlayer becomes successively more and more stiff, we will have $\gamma_x \rightarrow \gamma$, and boundary condition i) becomes identical to the one in part 1.5.3.

We may now write the equation system as:

$$\begin{bmatrix} -1 & +1 & (1+j(\alpha-\gamma_x)) & (-1+(\alpha-\gamma_x)) \\ -j & +1 & (-j+\delta) & (1+\delta) \\ (1+j\beta) & (1-\beta) & -1 & -1 \\ (j-\nu) & (1+\nu) & +j & +1 \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} +1 \\ -j \\ -1+j\beta \\ j+\nu \end{bmatrix} \quad (28)$$

This gives the following expressions:

$$r = \frac{R_r + jR_i}{N_r + jN_i} \quad (29)$$

$$r_j = \frac{R_{jr} + jR_{ji}}{N_r + jN_i} \quad (30)$$

$$t = \frac{jT_i}{N_r + jN_i} \quad (31)$$

$$t_j = \frac{T_{jr} + jT_{ji}}{N_r + jN_i} \quad (32)$$

where:

$$N_r = (4v+4\beta) + \delta(4+4v-2v\beta) + (\alpha-\gamma_x)\{(4-4\beta-2v\beta) + \delta(-2v-2\beta)\}$$

$$N_i = (-16-4v+4\beta+2v\beta) + \delta(-4+6\beta+2v\beta) + (\alpha-\gamma_x)\{(4+6v-2v\beta) + \delta(2+2v-2\beta-2v\beta)\}$$

$$R_r = (-4v+4\beta+2v\beta) + \delta(-4-4v+2\beta+2v\beta) + (\alpha-\gamma_x)\{(4+2v-4\beta-2v\beta) + \delta(2+2v-2\beta-2v\beta)\}$$

$$R_i = \delta(4\beta+2v\beta) + (\alpha-\gamma_x)\{(-4v+2v\beta) + \delta(-2v-2\beta)\}$$

$$R_{jr} = (4\beta+2v\beta) + \delta 2\beta + (\alpha-\gamma_x)\{-4+2v-4v\beta + \delta(-2-2v\beta)\}$$

$$R_{ji} = (-4v+2v\beta) + \delta(4+2\beta+4v\beta) + (\alpha-\gamma_x)\{2v-2\delta-2v\beta\delta\}$$

$$T_i = (-16-4v+4\beta) + \delta(-4+4\beta) + (\alpha-\gamma_x)\{4+4v\}$$

$$T_{jr} = -4\beta + (\alpha-\gamma_x)\{4+4v\}$$

$$T_{ji} = -4v + \delta(4-4\beta)$$

The expressions for r and t are identical to those in part 1.6.4 if γ is replaced by γ_x , but we have also

noted the expressions for t_j and r_j in order to enable the calculations of ρ_{BL} and τ_{BL} .

In analogy with the derivation in part 1.5.3, we have:

$$\tau = \tau_{BB} = |t|^2 \quad (33)$$

$$\rho = \rho_{BB} = |t|^2 \quad (34)$$

$$\begin{aligned} \tau_{BL} &= \frac{c_L}{2c_B} \left| \frac{v_{x_3}}{v_{y_1}} \right|^2 \\ &= \frac{c_L c_B}{2\omega^2 r_S^2} \left| \frac{\gamma_X}{\delta} \right|^2 |jt + t_j|^2 \end{aligned} \quad (35)$$

However the situation is not symmetric here, and $\rho_{BL} \neq \tau_{BL}$. Instead we must write:

$$\rho_{BL} = \tau_{BL} / \left| 1 + \frac{j\omega Z_L}{K_X} \right|^2 \quad (36)$$

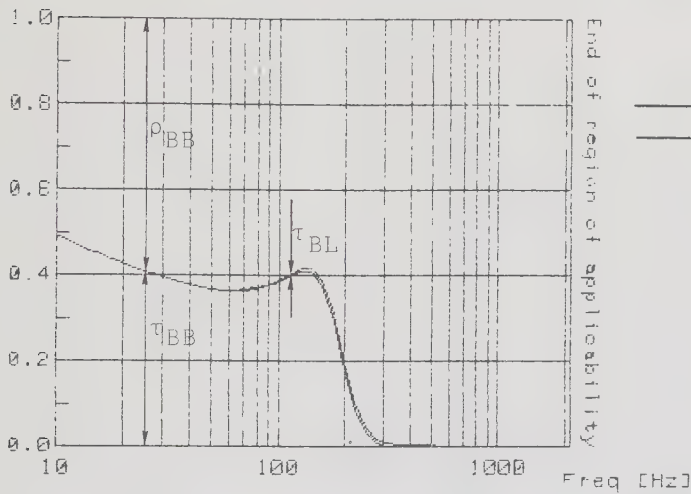
where we have considered boundary condition v) together with equation (24).

The situation is, however still symmetric in the sense, that if we move the blocking mass to the other side of the elastic interlayer, τ_{BB} and ρ_{BB} will remain unchanged, but ρ_{BL} will take over the present value of τ_{BL} from equation (35) and vice versa.

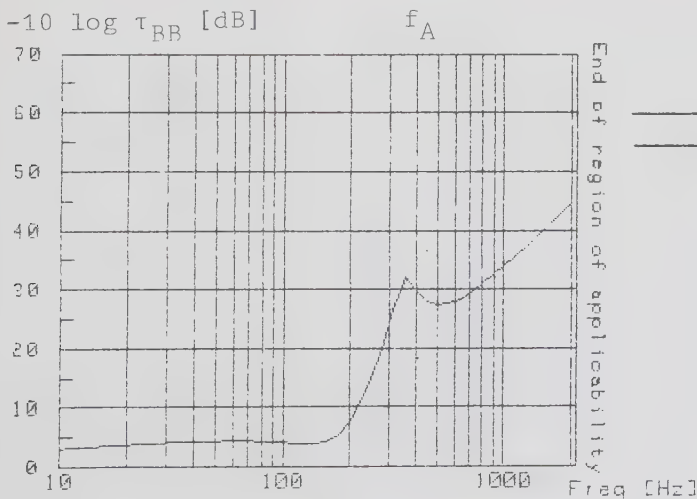
We would now like to show the calculations for the case we "imitated" in part 1.6.4, an excentric beam with $b_0 \times h_0 = 2h \times 2h$:

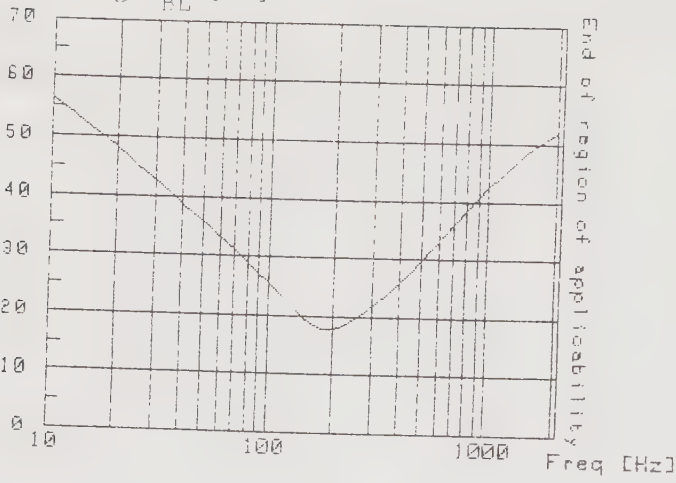
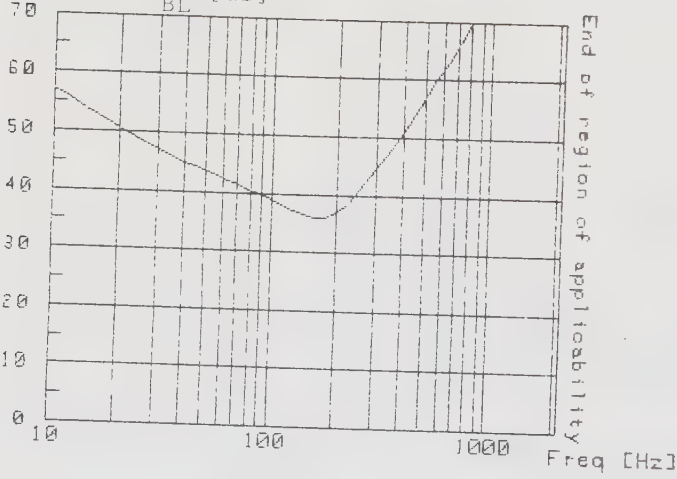
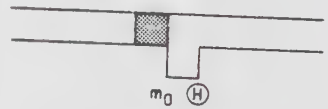
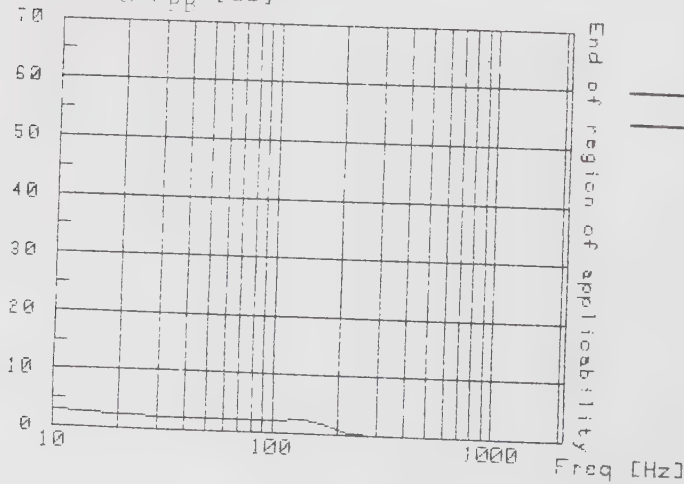
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m ³]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	
Excentric Blocking		1		
Mass	Bo x Ho [m]	0.160x0.160		
Extra Mass	Mo [kg/m]	59		
MassMom of Inert	[kgm]	1.1E+00		



Transmission and Reflection Efficiencies for a Bending Wave at a Junction with an Elastic Layer and an Excentric Blocking Mass on One Side [EBt]



$-10 \log \tau_{RL} \text{ [dB]}$

 $-10 \log \rho_{BL} \text{ [dB]}$

 $-10 \log \rho_{pp} \text{ [dB]}$


As before we are mainly interested in $-10 \log \tau_{BB}$, The other curves are shown more for the sake of completeness. We see that both τ_{BL} and especially ρ_{BL} are extremely low, and although they will be somewhat larger for a more excentric beam, their influence on τ_{BB} will only be marginal. We also see, if we compare the curve for $-10 \log \tau_{BB}$ above to the "imitated" case in part 1.6.4, that the curves can be said to be identical, except for the lower top at f_A here, than in 1.6.4.

1.7 Summary and concluding remarks to Part 1

In part 1.3 we first presented and discussed Cremer's solution of an elastic interlayer between two identical beams. We discussed the frequency variation of the transmission loss with its frequencies of total transmission, f_T , and total attenuation, f_A . We found for our chosen practical example, that we have f_T at about 160 Hz, and a very low transmission loss at frequencies below approximately 500 - 800 Hz. f_A was found to have a value of about 8000 Hz.

Next we investigated the influence of losses in the interlayer, and found that these can be neglected, except for frequencies near f_T . Even here the influence is only marginal, instead of zero transmission loss at f_T , we have 1 - 2 dB.

We also investigated the influence of different beams on each side of the interlayer. Changing the material or the cross section of one of the beams, lowers f_T and there we also get a transmission loss of a few dB instead of zero. Above f_T we get a few dB improvement as the curve as a whole has been displaced. However the total improvement is relatively small, even for the drastic change of doubling the cross section of one of the beams. This displaces the curve only about one 1/3 octave step. For the remainder of Part 1 we disregard the losses in the interlayer and consider only identical beams on each side of the junction.

In part 1.4 we first investigate what happens if rotation is prevented on one side or both sides of the interlayer. This extra restriction is seen to have very little influence on the transmission loss. We

then investigate the influence of preventing the lateral deflection on one side of the interlayer. This on the other hand improves the transmission loss very much, giving high values over the entire frequency region of interest. Only a marginal extra improvement is obtained by preventing the lateral deflection on the other side as well. By introducing these four different cases of ideal boundary conditions at the junction, we have shown clearly that it is the shear stiffness of the elastic interlayer which is critical. The elastic interlayer in our example is sufficiently "soft" to prevent transmission of the bending moments. However, the shear forces are transmitted rather too easily. If we were able to reduce the shear stiffness of the interlayer we would get a lower value of K and f_T according to equation (1.3.21). We would also have a lower value of f_A , and the result would be an improved transmission loss. No practical solution has yet been found to "tie down" the beam on one side of the interlayer. This too would improve the transmission loss greatly as discussed above.

We would now like to consider more practical constraints at the junction. In most cases there are lightweight walls on each side of the junction, and usually there are also stiffening beams to take up the extra static load from the walls. Thus we have both a restraint in lateral deflection because of the static load, and also a rotational restraint because of the extra mass moment of inertia of the edge beams.

In part 1.5 we consider the influence of blocking masses by themselves. This case has been studied by Cremer, and we present his solution from [1] and [2] for a symmetric blocking mass, and the effect of a

non-rigid connection to the beam. We also present his solution for the effect of a finite clamping length, which moves f_A to a higher frequency. This decreases the transmission loss at frequencies near the f_A for a point-attached blocking mass. Finally we consider an excentric blocking mass, as in practice the edge beams are of this type. We assume an incident bending wave and obtain a solution in agreement with Cremer's considerations for an incident longitudinal wave. The result is, that the generation of longitudinal waves influences very little on the numerical value of the transmission loss for bending waves. If we take a blocking mass and compare the result for a symmetric and an excentric placing, we get a lower f_A for the excentric case because of the larger mass moment of inertia. However if we compare the two different placings for equal m_0 and θ , we find that the difference is only marginal, except at f_A . In practical cases this difference at f_A is less because of losses etc. and our conclusion is that we can "imitate" the case of an excentric blocking mass by choosing a symmetric one with the same mass moment of inertia and the same mass. This would simplify the calculations. As for the effect of the finite clamping length, it seems as though this may have to be considered, at least if exact calculations are to be made. In more approximate calculations it may be sufficient to reduce the mass moment of inertia, which has a similar effect. A 50 % reduction may be an acceptable approximation for many of our practical examples.

In part 1.6 we consider "practical" constraints at the joint. At first we make calculations for an extra mass on one side of the elastic layer only. The "practical" situation would be a lightweight wall, where the elastic layer is not in the middle of the wall, but

on one side of it. (Perhaps not very practical!) We see here that for the relatively small extra mass 200 kg/m, f_A is lowered from about 8000 to about 700 - 800 Hz, and we get a considerable improvement over the whole frequency region of interest. Even at f_T we no longer have zero transmission loss, but almost 10 dB. The improvement would be even greater for a (more realistic) larger mass.

It is rather difficult to estimate the static load for a general case of lightweight wall, but 200 kg/m should not be an overestimated value.

Next we consider the more usual case, where the two parts of the lightweight wall are on each side of the elastic layer, 100 kg/m on each side. Here we get an even lower f_A , but we also have zero transmission loss at f_T .

The case of two unequal masses gives much the same result, and we continue to the case of a reinforcing edge beam in combination with an elastic layer. First we try the case of an edge beam on one side only. The result is similar to that of an extra mass on one side.

The frequency f_A is now primarily determined by the mass moment of inertia, and if we reduce this to account for the finite clamping length, f_A will be moved to a slightly higher frequency. This has very little influence on the curve on each side of f_A , the peak at $f = f_A$ is just moved a little. In practice we would not expect the extremely high values at f_A anyway, and the conclusion is, that disregarding the effect of a finite clamping length will not be a major source of error. Thus this correction is disregarded in the rest of the report. An extra mass, representing a lightweight wall is also here seen to improve the transmission loss.

The next step is to consider the more practical case of blocking masses on each side of the elastic layer. Here the result is somewhat disappointing. The frequency f_A is surely lowered, but as the elastic layer has a larger contact area with the beams on each side, we have a much lower transmission loss above f_A than in our previous examples. As in our previous symmetric case we also have a frequency of total transmission, f_T . Besides this relatively low level of the transmission loss there is yet another phenomenon that lowers this level. We have another frequency of total attenuation, which we denote as $f_{A\alpha}$, as it seems to depend most strongly on the value of α (the mass moment of inertia). We also seem to have a corresponding frequency region of large transmission, $f_{T\alpha}$, and this coincides with the frequency region of f_A , making the transmission loss there even lower. Adding extra masses is seen to have only a marginal effect here, and we have about the same situation as in our original case in part 1.3, where we have a very low transmission loss up to about 600 - 800 Hz.

However, if we were able to limit the contact area of the elastic layer to that of the original case, we would have a high transmission loss already at 200 Hz, similar to the case we had for a single sided blocking mass. However we still have the frequency of total transmission, f_T , which we do not have in the single sided case.

Finally we show the calculations for a single sided excentric blocking mass, where we investigate if the presence of the elastic layer changes anything in our previous assumption, that the case of an excentric blocking mass can be imitated by a symmetric one with the identical mass and the identical mass moment of inertia.

The result just confirms once again our assumption. There is only marginal difference between the two cases.

We intended to make calculations for excentric blocking masses on both sides as well, but this hardly seems motivated.

PART 2

TWO DIMENSIONAL
THEORETICAL STUDY
(PLATES)

2.1 Introduction

In the second part of this report we shall extend some of our solutions from Part 1 to include the two-dimensional problem of oblique incidence of bending waves in plates. The junctions or discontinuities are still elastic interlayers, which we assume to be thin and locally reacting.

Our first case is that of an elastic interlayer between two identical plates.

Secondly we consider the two-dimensional case of a reinforcing beam on a plate. Besides the extra mass moment of inertia we now also have the extra torsional stiffness to consider.

Thirdly we combine the elastic layer with a reinforcing beam or beams on one side or both sides.

Finally we have a short summary with concluding remarks to Part 2.

2.2 Theoretical model

Also here in Part 2 we use a similar theoretical model as used by Cremer in [1], [2]. We assume the plates to be semi-infinite and they lie in the xz -plane with the y -direction upwards. The junction is placed along the z -axis, and we assume the bending wave to arrive from ($x < 0$, $z > 0$) at an angle of incidence ϕ . The reflected wave has this same directional angle ϕ in the opposite direction. As we always assume that we have identical plates on each side of the interlayer, the transmitted wave also has this same directional angle ϕ .

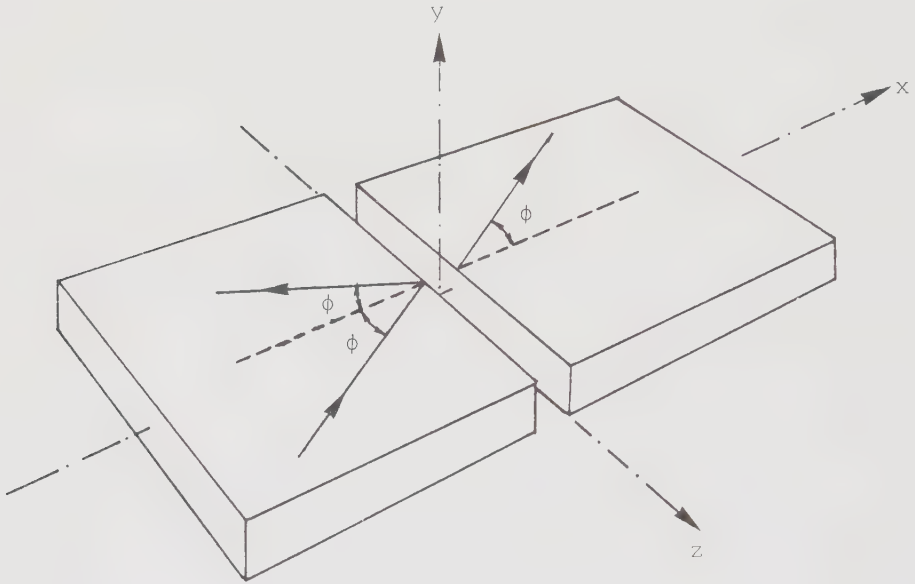


Figure 2.2.1. Configuration for oblique incidence.

We consider only bending waves here, as our blocking masses are symmetric and thus cannot generate in-plane waves at the junctions. As in Part 1 we only consider simple, or pure bending waves, but our field variables are a little bit more complicated in the two-dimensional plates.

As before we have the angular velocity

$$w_z = \frac{\partial v_y}{\partial x} \quad (1)$$

and the corresponding moment, which is now defined per unit width as:

$$M_{xz} = -\frac{B}{j\omega} \left(\frac{\partial^2 v_y}{\partial x^2} + \mu \frac{\partial^2 v_y}{\partial z^2} \right) \quad (2)$$

with

$$B = (1/12) h^3 E / (1 - \mu^2) \quad (3)$$

(For the two-dimensional field variables, see also [31].)

However we also have the rotation and moments along the boundary line to consider:

$$w_x = -\frac{\partial v_y}{\partial z} \quad (4)$$

$$M_{xx} = -\frac{B(1 - \mu)}{j\omega} \frac{\partial w_x}{\partial x} \quad (5)$$

Now $\frac{\partial M_{xx}}{\partial z}$ represents vertical forces per unit length, which can be added to the shear forces, and this gives:

$$F_{xy} = -\frac{\partial M_{xz}}{\partial x} - \frac{\partial M_{zz}}{\partial z} + \frac{\partial M_{xx}}{\partial z} \quad (6)$$

where $M_{zz} = -M_{xx}$, and we have:

$$F_{xy} = \frac{B}{j\omega} \left(\frac{\partial^3 v_y}{\partial x^3} + (2 - \mu) \frac{\partial^3 v_y}{\partial z^2 \partial x} \right) \quad (7)$$

In equations (1), (2) and (7) we now have our four field variables, w_z , M_{xz} and F_{xy} in terms of v_y , similar to Part 1.

The incident bending wave is written here as

$$v_{1+}(x) = v_{1+} e^{-jk_B x \cos \phi + jk_B z \sin \phi} \quad (8)$$

and in analogy to Part 1 we have reflected and transmitted bending waves and nearfields on each side of the junction

$$v_1(x) = v_{1+} (e^{-jk_B x \cos \phi} + r e^{-jk_B x \cos \phi} + r_j e^{k_B \sqrt{1+\sin^2 \phi} x}) \quad (9)$$

$$v_3(x) = v_{1+} (t e^{-jk_B x \cos \phi} + t_j e^{-k_B \sqrt{1+\sin^2 \phi} x}) \quad (10)$$

where we have omitted to write the factor $e^{jk_B z \sin \phi}$, which is common for all the terms in (9) and (10). We will also omit this factor in the future, but differentiation with respect to z will be accounted for by multiplying by the factor $jk_B \sin \phi$.

Our three equations (1), (2) and (7) then become:

$$w_z = \frac{\partial v_y}{\partial x} \quad (11)$$

$$M_{xz} = -\frac{B}{j\omega} \left(\frac{\partial^2 v_y}{\partial x^2} - \mu (k_B \sin \phi)^2 v_y \right) \quad (12)$$

$$F_{xy} = \frac{B}{j\omega} \left(\frac{\partial^3 v_y}{\partial x^3} - (2 - \mu)(k_B \sin \phi)^2 \frac{\partial v_y}{\partial x} \right) \quad (13)$$

The homogeneous bending wave equation is similar to the one for the one-dimensional case, but $\partial^4/\partial x^4$ has to be replaced by the quadrated Laplace operator

$$\nabla^4 = (\text{div grad})^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right)^2:$$

$$\nabla^4 v_y - k_B^4 v_y = 0 \quad (14)$$

Here k_B is the bending wavenumber

$$k_B^4 = \omega^2 m / B \quad (15)$$

with B according to equation (3) and $m = \rho h$ the mass of unit surface area.

Real values of k_B give a bending wave phase velocity

$$c_B = \sqrt{\omega} \cdot \sqrt{B/m} \quad (16)$$

and the group velocity

$$c_{Bg} = \frac{d\omega}{dk_B} = 2\sqrt{B/m} \cdot k_B = 2c_B \quad (17)$$

2.3 Elastic interlayer between two plates

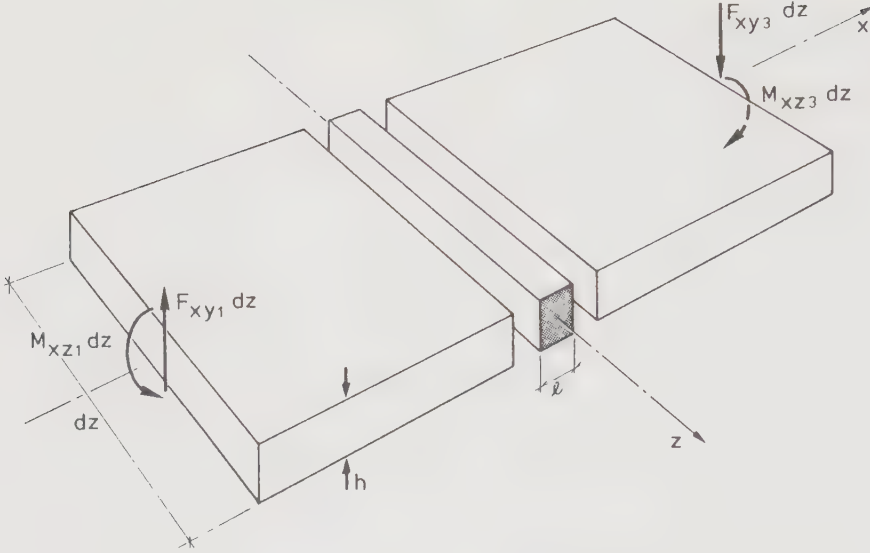


Fig 2.3.1. Configuration. We have left out the parameters w_z and v_y , but they are defined analogously to the one-dimensional case.

The boundary conditions here are the same as for the corresponding one-dimensional case:

- i) $M_{xz1} = M_{xz3}$
- ii) $F_{yx1} = F_{yx3}$
- iii) $(v_{y1} - v_{y3})/j\omega = F_{xy3}/K$
- iv) $(w_{z1} - w_{z3})/j\omega = M_{xz3}/C$

where the spring constants K and C are now defined per unit width:

$$K = \frac{h_2 G_2}{l} \quad (1)$$

$$C = \frac{h_2^3}{12} \frac{E_2}{(1 - \mu_2^2)} \quad (2)$$

The elastic layer is considered to be so thin, that it can be modelled as having the properties of a pure spring or a series of independent spring elements (locally reacting elastic layer).

We now use the expressions (2.2.9) and (2.2.10) for the wave fields on each side of the junction, and write all four boundary conditions in terms of v_y with the help of equations (2.2.11 - 2.2.13):

$$\begin{aligned} \text{i)} \quad & -(1-(1-\mu)\sin^2\phi) - r_j(1-(1-\mu)\sin^2\phi) + r_j(1+(1-\mu)\sin^2\phi) \\ & = -t(1-(1-\mu)\sin^2\phi) + t_j(1+(1-\mu)\sin^2\phi) \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad & j \cos\phi(1+(1-\mu)\sin^2\phi) - jr \cos\phi(1+(1-\mu)\sin^2\phi) \\ & + r_j \sqrt{1+\sin^2\phi}(1-(1-\mu)\sin^2\phi) \\ & = jt \cos\phi(1+(1-\mu)\sin^2\phi) - t_j \sqrt{1+\sin^2\phi}(1-(1-\mu)\sin^2\phi) \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad & 1+r_j - t - t_j = \beta \{ jt \cos\phi(1+(1-\mu)\sin^2\phi) \\ & - t_j \sqrt{1+\sin^2\phi}(1-(1-\mu)\sin^2\phi) \} \end{aligned}$$

$$\begin{aligned} \text{iv)} \quad & -j \cos\phi + jr \cos\phi + r_j \sqrt{1+\sin^2\phi} + jt \cos\phi + t_j \sqrt{1+\sin^2\phi} \\ & = v \{ t(1-(1-\mu)\sin^2\phi) - t_j(1+(1-\mu)\sin^2\phi) \} \end{aligned}$$

where we have introduced the help parameters β and v , defined as in Part 1:

$$\beta = \frac{\omega C_B^m}{K} \quad (3)$$

$$v = \frac{Bk_B}{C} \quad (4)$$

We obviously have to introduce further help parameters, and we have chosen the following:

$$\sigma = \sqrt{1 + \sin^2 \phi} \quad (5)$$

$$\psi = \sqrt{1 - \sin^2 \phi} = \cos \phi \quad (6)$$

$$\sigma_1 = 1 + (1 - \mu) \sin^2 \phi \quad (7)$$

$$\psi_1 = 1 - (1 - \mu) \sin^2 \phi \quad (8)$$

This enables us to write the equation system as:

$$\begin{bmatrix} -\psi_1 & \sigma_1 & \psi_1 & -\sigma_1 \\ -j\psi\sigma_1 & \sigma\psi_1 & -j\psi\sigma_1 & \sigma\psi_1 \\ 1 & 1 & -(1+j\beta\psi\sigma_1) & (-1+\beta\sigma\psi_1) \\ j\psi & \sigma & (j\psi-\nu\psi_1) & (\sigma+\nu\sigma_1) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} \psi_1 \\ -j\psi\sigma_1 \\ -1 \\ j\psi \end{bmatrix} \quad (9)$$

We solve this for the parameters r and t , and get the following analytical expressions:

$$r(\phi) = \frac{R_r}{N_r + jN_i} \quad (10)$$

$$t(\phi) = \frac{jT_i}{N_r + jN_i} \quad (11)$$

where

$$N_r = 4\nu \cdot (\sigma\psi_1^2) + 4\beta \cdot (\sigma\psi^2\sigma_1^2) + \nu\beta \cdot (\psi^2\sigma_1^4 - \sigma^2\psi_1^4)$$

$$N_i = -16 \cdot (\sigma\psi) - 4\nu \cdot (\psi\sigma_1^2) + 4\beta \cdot (\psi\sigma^2\psi_1^2) + 2\nu\beta \cdot (\sigma\psi\sigma_1^2\psi_1^2)$$

$$R_r = -4\nu \cdot (\sigma\psi_1^2) + 4\beta \cdot (\sigma\psi^2\sigma_1^2) + \nu\beta \cdot (\psi^2\sigma_1^4 + \sigma^2\psi_1^4)$$

$$T_i = -16 \cdot (\sigma\psi) - 4\nu \cdot (\psi\sigma_1^2) + 4\beta \cdot (\psi\sigma^2\psi_1^2)$$

We see that for normal incidence we have $\sigma = \psi = \sigma_1 = \psi_1 = 1$ and equations (10) and (11) reduce to those for the corresponding one-dimensional case (equations 1.3.13 - 1.3.14).

The condition for total transmission is $R_r = 0$, which can be written as:

$$4\nu \cdot (\sigma\psi_1^2) = 4\beta \cdot (\sigma\psi^2\sigma_1^2) + \nu\beta(\psi^2\sigma_1^4 + \sigma^2\psi_1^4) \quad (12)$$

From equations (5) - (8) we see that σ and σ_1 increase with increasing ϕ , but ψ and ψ_1 decrease. If we make the same simplifications as in part 1.3.1 we may write (12) as:

$$\beta \approx \frac{4\sigma\psi_1^2}{(\psi^2\sigma_1^4 + \sigma^2\psi_1^4)} \quad (13)$$

For normal incidence we have total transmission at $\beta \approx 2$ and at oblique incidence we have a numerical value less than two on the right hand side, which means a lower frequency for total transmission.

At grazing incidence, we have $\psi = 0$ and equation (10) reduces to:

$$r(90^\circ) = \frac{-4\nu\sigma\psi_1^2}{4\nu\sigma\psi_1^2} = -1 \quad (14)$$

which means that we have $\rho = 1$ and $\tau = 0$.

The condition for total attenuation is $T_i = 0$, which can be written as:

$$\beta(\psi\sigma^2\psi_1^2) = \nu(\psi\sigma_1^2) + 4(\sigma\psi) \quad (15)$$

Still making the same simplifications as in part 1.3.1 we may write (15) as

$$\beta \approx \nu(\sigma_1/(\sigma\psi_1))^2 \quad (16)$$

Now the angle parameter in (16) is larger than 1 for oblique incidence, and as β increases faster with frequency than ν , the condition (16) is fulfilled at a frequency higher than f_A for normal incidence.

We can now calculate the transmission and reflection efficiencies as a function of the angle of incidence. The intensity (power per unit length of the junction) is $I_B(\phi) = 2c_B \cdot m \cdot \tilde{v}_B^2 \cdot \cos \phi$ and we have:

$$\tau(\phi) = \frac{2c_B \cdot m \cdot \tilde{v}_{3+}^2 \cdot \cos \phi}{2c_B \cdot m \cdot \tilde{v}_{1+}^2 \cdot \cos \phi} = |t(\phi)|^2 \quad (17)$$

$$\rho(\phi) = |r(\phi)|^2 \quad (18)$$

We calculate the average values by integrating over all angles of incidence:

$$\begin{aligned} \tau_d &= \frac{\frac{1}{\pi} \int_{-\pi/2}^{\pi/2} 2c_B m_2 \tilde{v}_{1+}^2 |t(\phi)|^2 \cos \phi \, d\phi}{\frac{1}{\pi} \int_{-\pi/2}^{\pi/2} 2c_B m_2 \tilde{v}_{1+}^2 \cos \phi \, d\phi} \\ &= \int_0^{\pi/2} |t(\phi)|^2 \cos \phi \, d\phi \end{aligned} \quad (19)$$

Correspondingly we get for ρ :

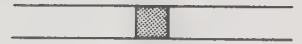
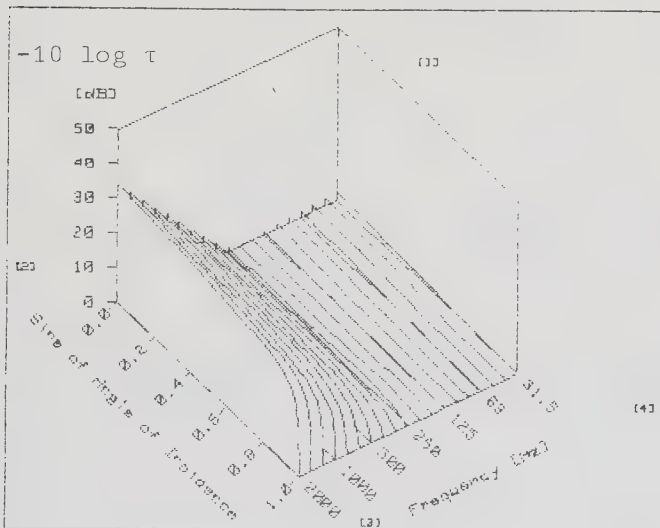
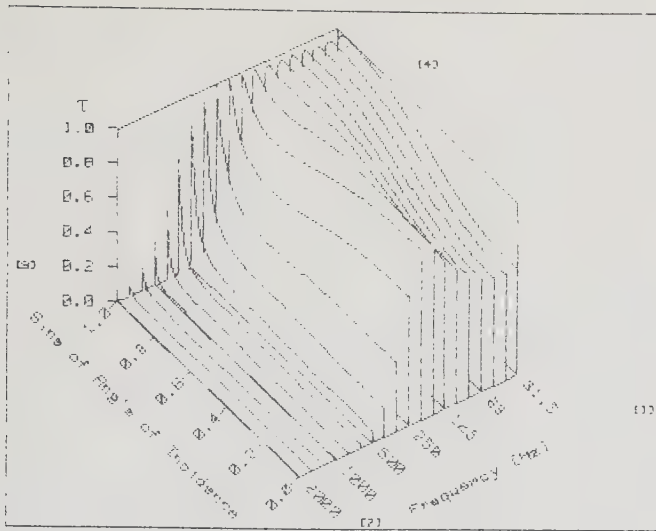
$$\rho_d = \int_0^{\pi/2} |r(\phi)|^2 \cos \phi \, d\phi \quad (20)$$

We will now take the same example as used in Part 1, and show the variation of τ with the frequency and with the angle of incidence. As no losses are included we have $\rho = 1 - \tau$.

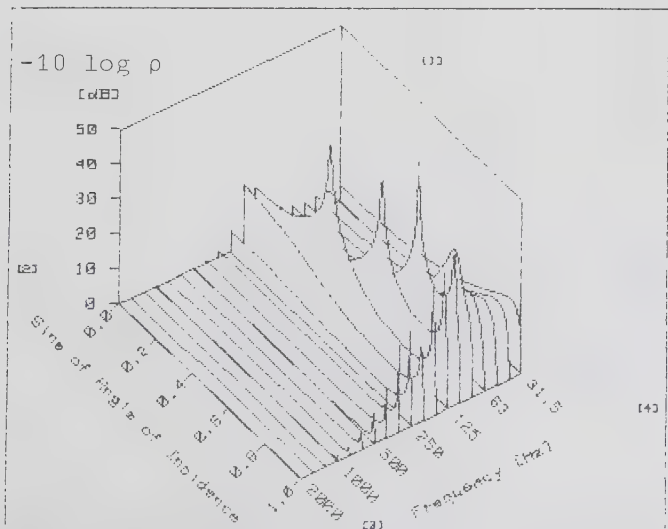
We also show the same type of figure for the transmission loss $-10 \log \tau$, and for $-10 \log \rho$.

MATERIALS AND DIMENSIONS

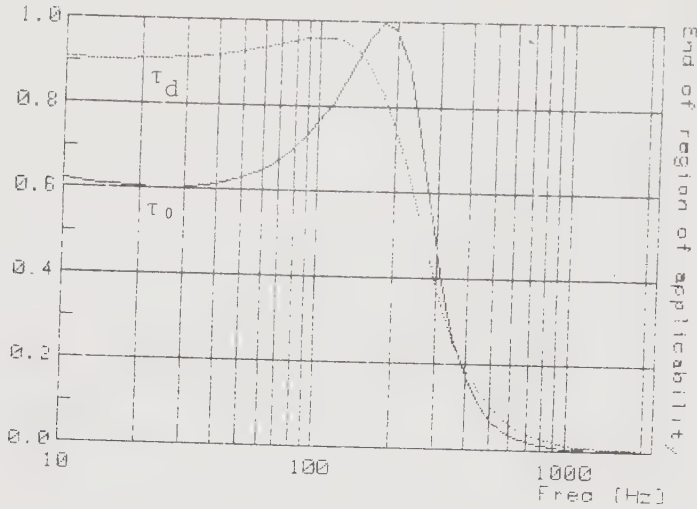
ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.5E+10	1.8E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m3]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	



Note the
different
viewpoint!

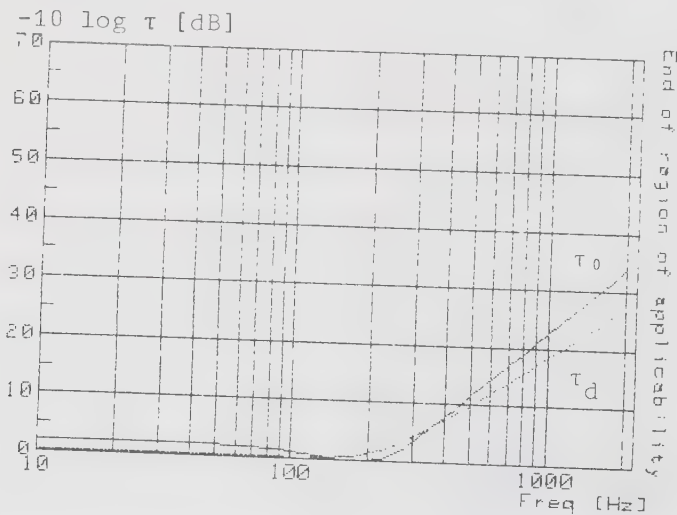


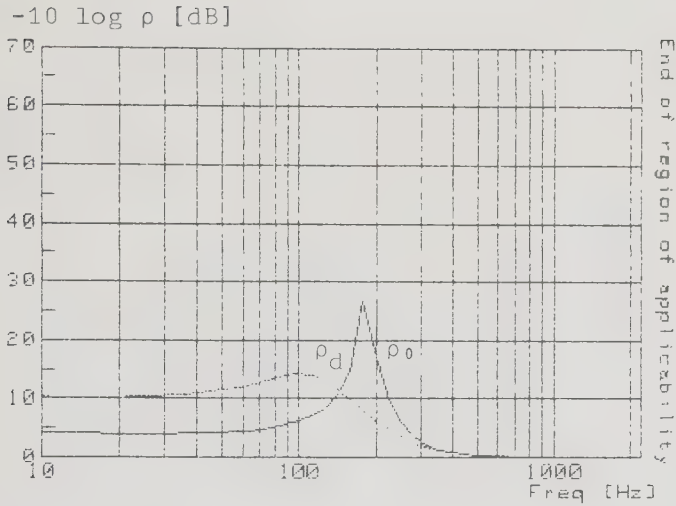
Finally we compare the frequency variation of τ_d and ρ_d , according to equations (19) and (20) to the corresponding values for normal incidence (τ_0 , ρ_0).



Transmission Efficiency for a Bending Wave
at a Junction with an Elastic Layer
Two Dimensional (Plates) [E2]

We also compare the logarithmic values $-10 \log \tau_d$ and $-10 \log \tau_0$, and finally $-10 \log \rho_d$ and $-10 \log \rho_0$.





Already from the 3-dimensional curve for $-10 \log \tau$ we can see that the angle variation of the transmission loss is very moderate. The transmission loss at all angles is rather similar to the one for normal incidence. This is confirmed by the diagram where we compare $-10 \log \tau_d$ and $-10 \log \tau_0$. The main changes from the one-dimensional case are that the frequency of maximum transmission is lowered, and below this, the transmission loss is close to zero. Above f_T the slope of the curve is not as steep as before, giving about 5 dB lower transmission loss at 2000 Hz for our particular example.

If we consider the 3-dimensional plot of $-10 \log \rho$, we see that f_T occurs at more and more oblique incidence for successively lower frequencies as we predicted (see the discussion below eq (13)). This means that the top at f_T is smoothed out for $-10 \log \rho_d$ as we can see in the figure above, and we may also note, that the maximum value occurs at a frequency almost an octave lower than f_T for normal incidence.

2.4 Reinforcing beam on a plate

(This case is analyzed by Cremer in [1],[2].)

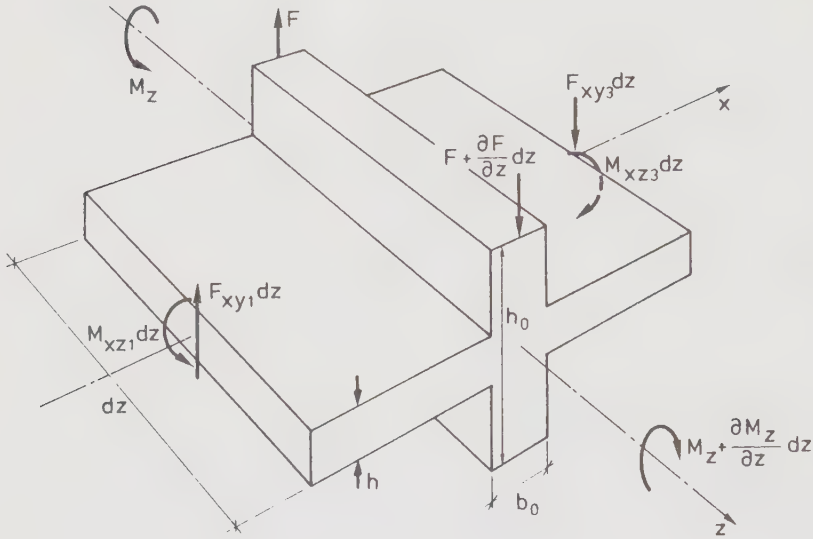


Figure 2.4.1. Configuration. We have left out the parameters v_y and w_z , which are defined analogously to the one-dimensional case.

The reinforcing beam is assumed to be rigidly attached to the plate and has a rectangular cross section (height $\times$ width = $h_0 \times b_0$). It has the mass m_0 per unit length, and the mass moment of inertia Θ per unit length. It also has the torsional stiffness T . These parameters are given by the following equations:

$$m_0 = \rho \cdot b_0 \cdot (h_0 - h) \quad (1)$$

$$\Theta = \rho(b_0 h_0^3 + h_0 b_0^3)/12 \quad (2)$$

$$T \approx \frac{1}{3} G b_0^3 h_0 \left(1 - \frac{192}{\pi^5} \left(\frac{b_0}{h_0} \tanh\left(\frac{\pi}{2} \frac{h_0}{b_0}\right)\right)\right) \quad (3)$$

The expression for T is due to Timoshenko [31].
(We have only taken the first term in a rapidly converging series.)

The beam also has a bending stiffness:

$$B_0 = (1/12) b_0 h_0^3 E \quad (4)$$

and a corresponding bending wave velocity

$$c_{B_0} = \sqrt{\omega^{-4} \sqrt{B_0 / m_0}} \quad (5)$$

The torsional stiffness and the mass moment of inertia have also a corresponding torsional wave velocity:

$$c_{T_0} = \sqrt{T / \Theta} \quad (6)$$

Now we can start to consider the boundary conditions. We may still assume that v_y is equal on both sides of the beam, and the same is true for w_z . The boundary condition for the moments becomes a little bit more complicated than for the one-dimensional case. We still have to make a correction for the angular acceleration of the blocking mass, but in addition we must consider the lengthwise rate of change of the torsional moment M_z (see fig 2.4.1). The moment equilibrium at the junction is thus:

$$M_{xz_1} - M_{xz_3} = j\omega\Theta w_{z_3} + \frac{\partial M_z}{\partial z} \quad (7)$$

Similarly it is not sufficient to make a correction for the acceleration of the mass in the boundary condition for the edge forces. The beam bends in the yz -plane, and this results in a lateral shear force, F , with a rate of change that also must be included in the force equilibrium at the junction:

$$F_{xy_1} - F_{xy_3} = j\omega m_0 v_{y_3} + \frac{\partial F}{\partial z} \quad (8)$$

If we consider that $M_Z = -(T/j\omega) \partial w_{Z_3} / \partial z$ and $F = (B_0/j\omega) \partial^3 v_y / \partial z^3$ we may now write our four boundary conditions as:

- i) $M_{xz_1} - M_{xz_3} = j(\omega\theta - (k_B \sin \theta)^2 T/\omega) w_{z_3}$
- ii) $F_{xy_1} - F_{xy_3} = j(\omega m_0 - (k_B \sin \theta)^4 B_0/\omega) v_{y_3}$
- iii) $v_{y_1} = v_{y_3}$
- iv) $w_{z_1} = w_{z_3}$

As pointed out by Cremer we have a possible trace matching in both i) and ii), when the right hand sides become zero. This means a total transmission of the moments and forces respectively. The condition in i) is frequency dependent, and cannot occur above a certain upper frequency limit. The trace matching condition can be written as:

$$c_B = c_{T_0} \sin \phi \quad (9)$$

and for frequencies where $c_B > c_{T_0}$, this condition cannot be fulfilled for any angle of incidence. The condition in ii) can be written as:

$$c_B = c_{B_0} \sin \phi \quad (10)$$

and as the two bending wave velocities have the same frequency dependence, this trace matching occurs at the same angle of incidence for all frequencies.

We may now introduce help variables corresponding to those we used in the one-dimensional case of a blocking mass:

$$\alpha = \frac{\omega^2 \theta}{B k_B} \left(1 - \left(\frac{c_{T_0}}{c_B \sin \phi} \right)^2 \right) \quad (11)$$

$$\delta = \frac{m_0 \omega}{m c_B} \left(1 - \left(\frac{c_{B_0}}{c_B \sin \phi} \right)^2 \right) \quad (12)$$

By using these and the same help variables as in part 2.3, we may now write the equation system as:

$$\begin{bmatrix} -\psi_1 & \sigma_1 & (\psi_1 + j\alpha\psi) & (-\sigma_1 + \alpha\sigma) \\ -j\psi\sigma_1 & \sigma\psi_1 & (-j\psi\sigma_1 + \delta) & (\sigma\psi_1 + \delta) \\ 1 & 1 & -1 & -1 \\ j\psi & \sigma & j\psi & \sigma \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} \psi_1 \\ -j\psi\sigma_1 \\ -1 \\ j\psi \end{bmatrix} \quad (13)$$

Solving this equation to obtain the parameters r and t gives:

$$r(\phi) = \frac{R_r}{N_r + jN_i} \quad (14)$$

$$t(\phi) = \frac{jT_i}{N_r + jN_i} \quad (15)$$

where

$$N_r = 4\delta \cdot \sigma + 4\alpha(\sigma\psi^2) + \alpha\delta(\psi^2 - \sigma^2)$$

$$N_i = -16(\sigma\psi) - 4\delta \cdot \psi + 4\alpha(\sigma^2\psi) + 2\alpha\delta(\sigma\psi)$$

$$R_r = -4\delta \cdot \sigma + 4\alpha(\sigma\psi^2) + 2\alpha\delta$$

$$T_i = -16\sigma\psi - 4\delta\psi + 4\alpha\sigma^2\psi$$

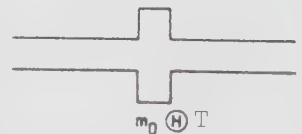
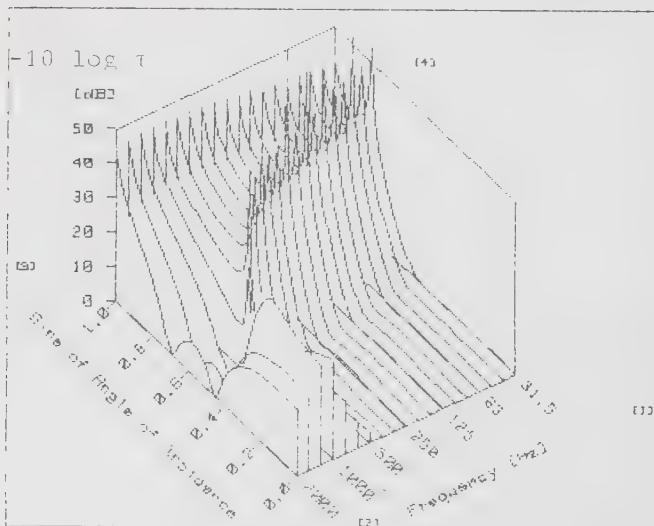
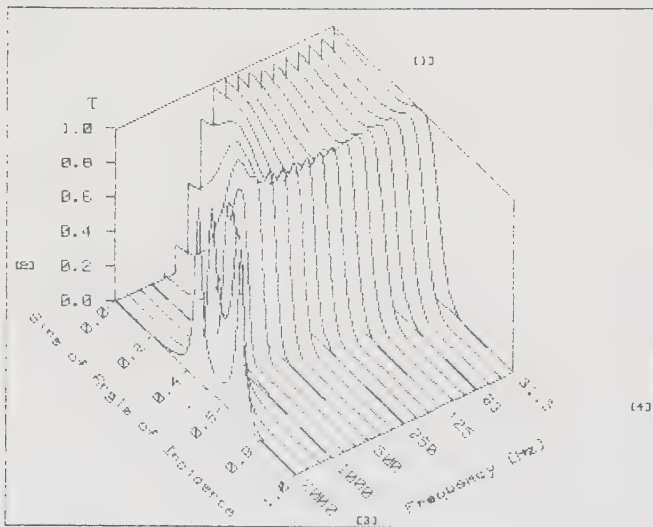
(Note that all the parameters σ_1 and ψ_1 have vanished, as they only appear as a sum: $\sigma_1 + \psi_1 = 2$.)

For normal incidence we see that the equations (14) and (15) are reduced to those for the one-dimensional case (eq. 1.5.7 - 8).

The values for $\tau(\phi)$, $\rho(\phi)$ and τ_d , ρ_d are defined as in part 2.3, and below we show the variation of τ with the frequency and with the angle of incidence. We have chosen as an example the same beam as we used in part 1.5.1.

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10		2.6E+10
Density	Rho[kg/m3]	2300		2300
Poiss. ratio	My [1]	0.25		0.25
Thickness	H [m]	0.080		0.080
Symmetric Blocking				
Mass	Bo x Ho [m]		0.160 x 0.400	
Density	Rho[kg/m3]		2300	
E-Modulus	E[Pa]		2.6E+10	
G-Modulus	G[Pa]		1.0E+10	
Extra Mass	Mo [kg/m]		118	
MassMom of Inert [kgm]			2.3E+00	
Torsional Stiffn.[Nm2]			4.3E+06	



Note the different viewpoint!

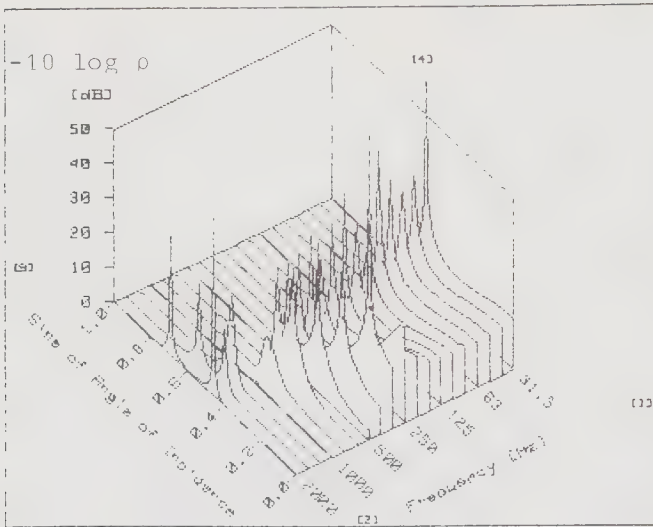
The linear presentation of τ seems to agree well with Cremer's interpretation of expressions (14) and (15), which he writes with the help of somewhat different parameters. (The solutions have been shown to be identical.)

When we turn to the logarithmic presentation, $-10 \log \tau$, we may first point out that we have chosen a different viewpoint in order to see the "landscape" more clearly. We see a high transmission loss for all frequencies at grazing incidence, and we see a maximum at about 1000 Hz for normal incidence as pointed out by Cremer. However, we also see that this phenomenon is not specific for normal incidence, in fact it occurs at all frequencies below f_A for normal incidence, but at different angles of incidence. The attenuation occurs at successively more grazing incidence for a decreasing frequency. This phenomenon is in accordance with the vanishing of the numerator in (15), which we can write as:

$$-4\sigma - \delta + \alpha\sigma^2 = 0 \quad (16)$$

For normal incidence we have $\alpha - \delta = 4$, and as $\sigma > 1$ and it increases with an increasing angle of incidence, while both α and δ decrease with decreasing frequency, we have a possibility for fulfillment of (16) by lowering the frequency and increasing the angle of incidence. We may also point out that α and δ themselves depend on the angle of incidence according to equations (11) and (12) and decrease as ϕ increases. Because of his choice of help parameters, this phenomenon was difficult for Cremer to discover, and in his discussions he treats it as a purely normal incidence phenomenon.

To see more clearly the two phenomena of total transmission, we have also plotted the 3-dimensional curve for $-10 \log \rho$ below.

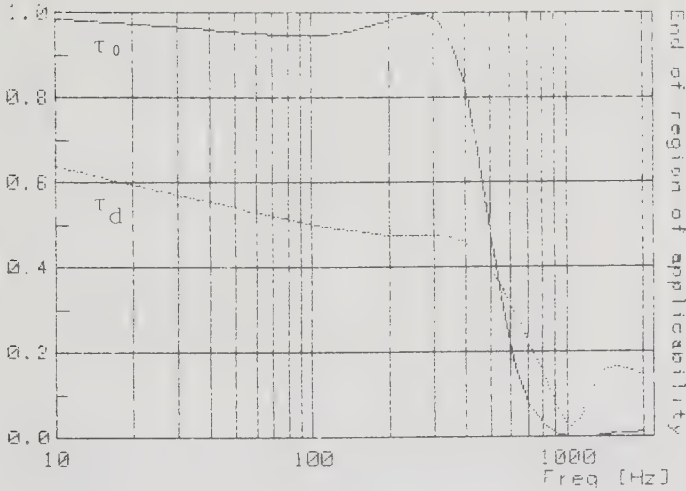


We see one row of peaks, which always occurs at the same angle of incidence, approximately at $\arcsin(0.45)$. This is the trace matching phenomenon of the two bending wave velocities in equation (10), which gives us the numerical value $\sin \phi = c_B/c_{B_0} \approx \sqrt{h/h_0} = 0.447$.

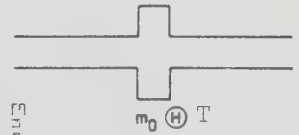
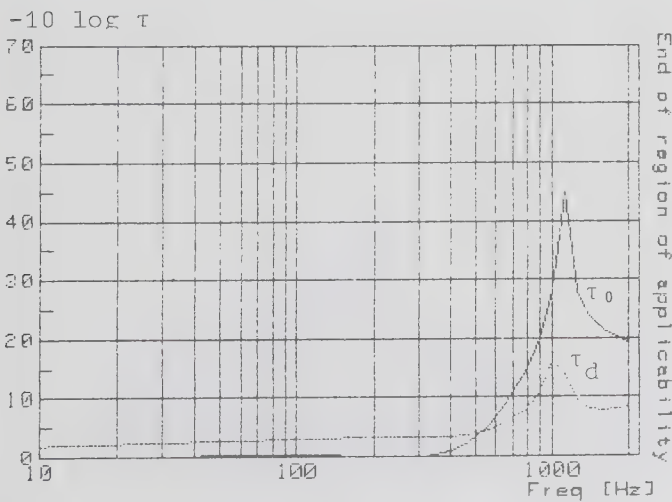
We also have the trace matching from equation (9) to consider, which occurs at different angles for different frequencies. At low frequencies this angle is almost zero, and the phenomenon does not exist for normal incidence, and apparently neither for "almost normal" incidence. The incident wave must have a "minimum obliqueness" for the trace matching to develop. It is first seen at about 250 Hz, where equation (9) gives us $\sin \phi = 0.26$ or $\phi = 15^\circ$. At 2000 Hz we get $\sin \phi = 0.72$ from equation (9) which agrees well with the figure, and at about 3800 Hz we would not have any angle

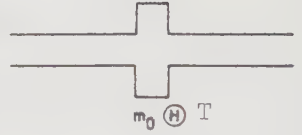
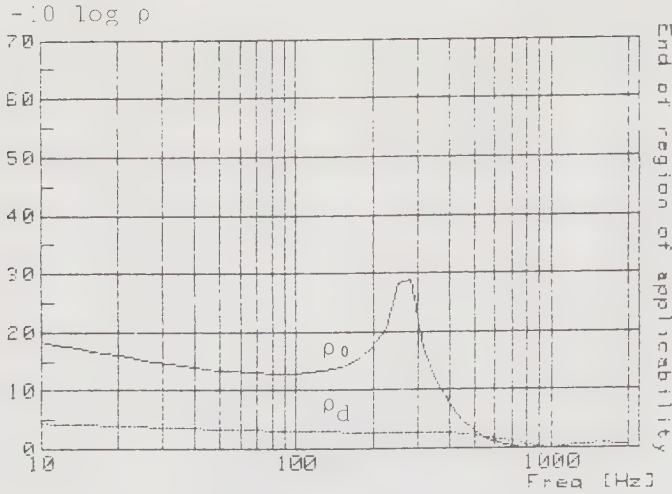
satisfying equation (9). [However, we would then be beyond the frequency limit for the applicability of simple bending wave theory.]

Finally we compare the frequency variation of τ_d and ρ_d to the corresponding case of τ_0 and ρ_0 .



Transmission Efficiency for a Bending Wave
at a Junction with a Symmetric Beam
Two Dimensional (Plates) [Bx2]





We may point out, that instead of zero or almost zero transmission loss at low frequencies, we now have a value of a few dB. The main change is however that the peak at f_A for normal incidence is very much lower for the diffuse case, and so are the values on each side of this peak. The maximum value also occurs at a slightly lower frequency for the diffuse case than for normal incidence.

We might be concerned about the validity of our simple bending wave theory in the case of the reinforcing beam. In our case of concrete slabs and concrete beams, which may have a height of 2 - 5 times the thickness of the slabs, the simple bending wave theory becomes unapplicable already at relatively low frequencies. For a concrete beam we have this limiting frequency approximately as:

$$f < \frac{170}{h} \quad (17)$$

Above this frequency we should use corrected bending waves. In [32] a simple correction method is discussed,

where c_{B_0} asymptotically assumes the value of c_S , the shear wave velocity at high frequencies. The transition from c_{B_0} to c_S is simply assumed to be:

$$c'_{B_0} = (c_{B_0}^{-4} + c_S^{-4})^{-1/4} \quad (18)$$

where the shear wave velocity is:

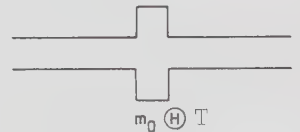
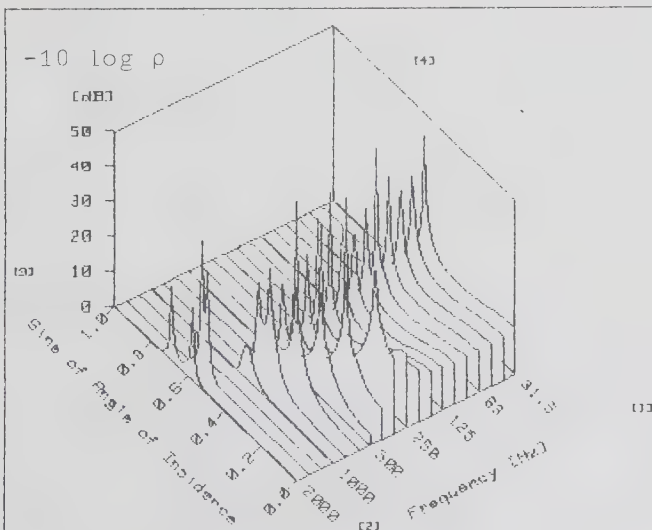
$$c_S = \sqrt{G/\rho} \quad (19)$$

The frequency variation of c'_{B_0} is shown to agree rather well with the exact solution for corrected bending waves. We may make this correction by introducing a "corrected" bending stiffness for the reinforcing beam:

$$\begin{aligned} B'_0 &= B_0 / (1 + (c_{B_0}/c_S)^4) \\ &= \frac{B_0}{1 + \omega^2 \rho h_0^2 (1 + \mu)^2 / (3E)} \end{aligned} \quad (20)$$

Among other things this leads to a slight frequency dependence for the trace matching condition ii). (See equation (10).)

This is most clearly seen in the figure for $-10 \log \rho$, which we have plotted below taking into account the corrected bending stiffness.



Note: Corrected bending stiffness B'_0 .

The deviations from our previous figure are only marginal, but we see that the trace matching ii) at 2000 Hz now occurs at $\sin \theta = 0.6$ instead of previously $\sin \theta = 0.45$. The effect on the diffuse values, τ_d and ρ_d , are very marginal indeed. The only noticeable difference is that we now have the maximum value for $-10 \log \tau_d$ at the same frequency as for the case of normal incidence. However, the numerical value hardly changes at all, and we disregard this correction for the rest of our cases.

We will not try to introduce corresponding "corrected bending waves" for the slabs. Our practical examples are thin slabs (approximately 80 mm) and the simple bending wave theory is applicable up to about 2000 Hz. We consider this sufficient here, especially as the phenomenon of large transmission is mainly a problem at low or intermediate frequencies.

In Part 1 of this report we discussed the fact that the practical edge beams in our case are usually not symmetric, but excentric. We made some calculations for excentric blocking masses, and found out, that the generation of longitudinal waves at the junction affected the transmission loss for the bending waves very little. We may try to make similar calculations here, for an excentric reinforcing beam. Instead of Z_L we could use Z_{LT} deduced in [14], the impedance of in-plane waves (both longitudinal waves and transverse waves). We could calculate the mass moment of inertia in a similar way as before, but no suitable model has been found for the torsional stiffness of such a beam. From handbooks in statics it can be seen, that the difference between the symmetric beam and the excentric beam with equal dimensions is not very great, it may be a question of 15 - 20 % for our practical configurations.

We have chosen to confine our two-dimensional calculations to a symmetric beam only. It can be assumed to have the same mass and the same mass moment of inertia as the practical excentric beam. The torsional stiffness will then be calculated as if the excentric beam were symmetrical, and this value is used in the calculations. The beam height used in calculating the torsional stiffness is the total height i.e. the extra height of the excentric beam + the height of the slab.

The error we make by using a symmetric beam instead of the real, excentric one, may be a little larger here than in our one-dimensional case. However, the calculations may probably still be considered to be a fairly correct estimate of the bending wave transmission loss.

2.5 Combination of an elastic layer and reinforcing beams

2.5.1 Symmetric beam on one side of the elastic layer

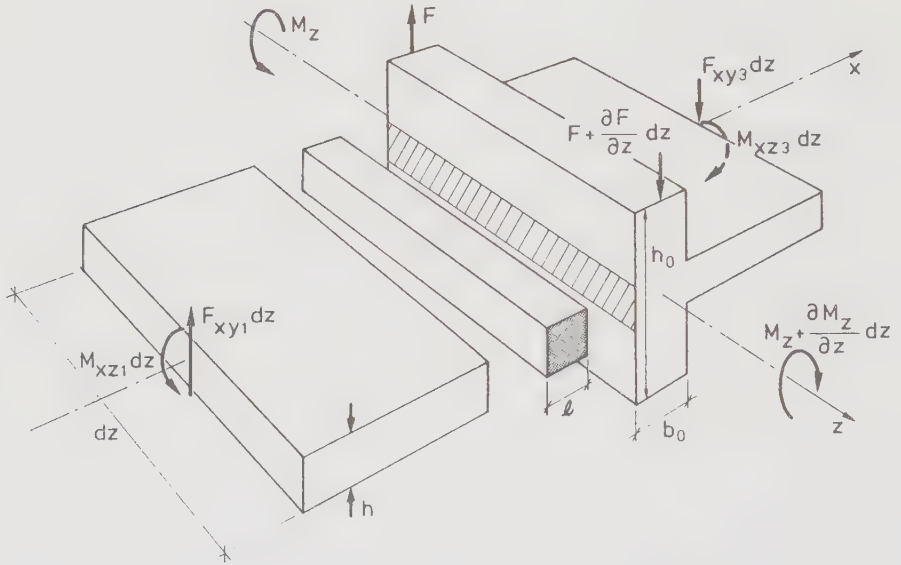


Figure 2.5.1. Configuration. The parameters v_y and v_z are not shown, but they are defined analogously as before.

We now consider the case of a reinforcing beam on one side of the elastic layer. As in Part 1 we disregard the finite width of both the beam and the elastic layer, and get the following boundary conditions in analogy with parts 2.3 and 2.4:

- i) $M_{xz_1} - M_{xz_3} = j\omega(\Theta - Tk_B^2 \sin^2 \phi / \omega^2) w_{z_3}$
 ii) $F_{xy_1} - F_{xy_3} = j\omega(m_0 - B_0 k_B^4 \sin^4 \phi / \omega^2) v_{y_3}$
 iii) $v_{y_1} - v_{y_3} = (j\omega/K) F_{xy_1}$
 iv) $w_{z_1} - w_{z_3} = (j\omega/C) M_{xz_1}$

The equation system with the same help parameters as before can be written as:

$$\begin{bmatrix} -\psi_1 & +\sigma_1 & (\psi_1 + j\alpha\psi) & (-\sigma_1 + \alpha\sigma) \\ -j\psi\sigma_1 & +\sigma\psi_1 & (-j\psi\sigma_1 + \delta) & (\sigma\psi_1 + \delta) \\ (1 + j\beta\psi\sigma_1) & (1 - \beta\sigma\psi_1) & -1 & -1 \\ (j\psi - \nu\psi_1) & (\sigma + \nu\sigma_1) & +j\psi & +\sigma \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} \psi_1 \\ -j\psi\sigma_1 \\ -1 + j\beta\psi\sigma_1 \\ j\psi + \nu\psi_1 \end{bmatrix} \quad (1)$$

The expressions for $t(\phi)$ and $r(\phi)$ become rather tedious here, and they are difficult to interpret, but we still write them out for the sake of completeness:

$$r(\phi) = \frac{R_r + jR_i}{N_r + jN_i} \quad (2)$$

$$t(\phi) = \frac{jT_i}{N_r + jN_i}$$

where

$$\begin{aligned} N_r = & 4\nu(\sigma\psi_1^2) + 4\beta(\sigma\psi^2\sigma_1^2) + \nu\beta(\psi^2\sigma_1^4 - \sigma^2\psi_1^4) \\ & + \delta\{4\sigma + 4\nu + \beta(\psi^2\sigma_1^2 - \sigma^2\psi_1^2) - 2\nu\beta(\sigma\psi_1^2)\} \\ & + \alpha\{4(\sigma\psi^2) + \nu(\psi^2\sigma_1^2 - \sigma^2\psi_1^2) - 4\beta(\sigma^2\psi^2) - 2\nu\beta(\sigma\psi^2\sigma_1^2)\} \\ & + \alpha\delta\{(\psi^2 - \sigma^2) - 2\nu\sigma - 2\beta(\sigma\psi^2) + \nu\beta(\sigma^2\psi_1^2 - \psi^2\sigma_1^2)\} \end{aligned}$$

$$\begin{aligned} N_i = & -16(\sigma\psi) - 4\nu(\psi\sigma_1^2) + 4\beta(\sigma^2\psi\psi_1^2) + 2\nu\beta(\sigma\psi\sigma_1^2\psi_1^2) \\ & + \delta\{-4\sigma + \beta(\sigma\psi(4 + \sigma_1^2 + \psi_1^2)) + 2\nu\beta(\psi\sigma_1^2)\} \\ & + \alpha\{4(\sigma^2\psi) + \nu(\sigma\psi(4 + \sigma_1^2 + \psi_1^2)) - 2\nu\beta(\sigma^2\psi\psi_1^2)\} \\ & + \alpha\delta\{2(\sigma\psi) + 2\nu\sigma - 2\beta(\sigma^2\psi) - \nu\beta(\sigma\psi(\sigma_1^2 + \psi_1^2))\} \end{aligned}$$

$$\begin{aligned}
R_r = & -4\nu(\sigma\psi_1^2) + 4\beta(\sigma\psi^2\sigma_1^2) + \nu\beta(\sigma^2\psi_1^4 + \psi^2\sigma_1^4) \\
& + \delta\{-4\cdot\sigma - 4\nu + \beta(\sigma^2\psi_1^2 + \psi^2\sigma_1^2) + 2\nu\beta(\sigma\psi_1^2)\} \\
& + \alpha\{4(\sigma\psi^2) + \nu(\sigma^2\psi_1^2 + \psi^2\sigma_1^2) - 4\beta(\sigma^2\psi^2) - 2\nu\beta(\sigma\psi^2\sigma_1^2)\} \\
& + \alpha\delta\{2 + 2\nu\cdot\sigma - 2\beta(\sigma\psi^2) - \nu\beta(\sigma^2\psi_1^2 + \psi^2\sigma_1^2)\}
\end{aligned}$$

$$\begin{aligned}
R_i = & \delta\{\beta(\sigma\psi(4 + \sigma_1^2 - \psi_1^2)) + 2\nu\beta(\psi\sigma_1^2)\} \\
& + \alpha\{-\nu(\sigma\psi(4 + \psi_1^2 - \sigma_1^2)) + 2\nu\beta(\sigma^2\psi\psi_1^2)\} \\
& + \alpha\delta\{-2\nu\cdot\psi - 2\beta(\sigma^2\psi) + \nu\beta(\sigma\psi(\psi_1^2 - \sigma_1^2))\}
\end{aligned}$$

$$\begin{aligned}
T_i = & -16(\sigma\psi) - 4\nu(\psi\sigma_1^2) + 4\beta(\sigma^2\psi\psi_1^2) \\
& + \delta\{-4\cdot\psi + 4\beta(\sigma\psi\psi_1)\} + \alpha\{4(\sigma^2\psi) + 4\nu(\sigma\psi\sigma_1)\}
\end{aligned}$$

For normal incidence these equations reduce to the ones for the corresponding one-dimensional case. In part 1.6.4, we have however happened to place the blocking mass on the other side of the elastic layer, and this gives the opposite sign on R_i above. (This of course does not affect the value of $|r|$ or ρ .)

In practice it may be better to solve (1) numerically for each combination of angle and frequency, as the analytical expressions for $r(\phi)$ and $t(\phi)$ are rather complex. This also reduces the risk of programming errors, especially if a standard routine is used for inverting the 4×4 matrix. We have, however, chosen to program the analytical expressions (2) and (3), and as a control we check if $\rho + \tau = 1$.

Now, the analysis above has been made for a symmetric beam, rigidly connected to the slab, and thus we imply that equations (2.4.1-2.4.3) should be used to calculate the parameters m_0 , θ and T . We are however not confined to these equations, and the analysis is of course valid for a different choice of these parameters. If we want to calculate the effect of only a light-weight wall on one side of the elastic layer, we

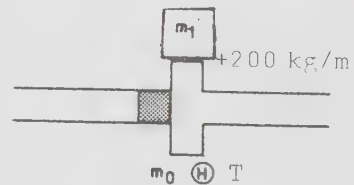
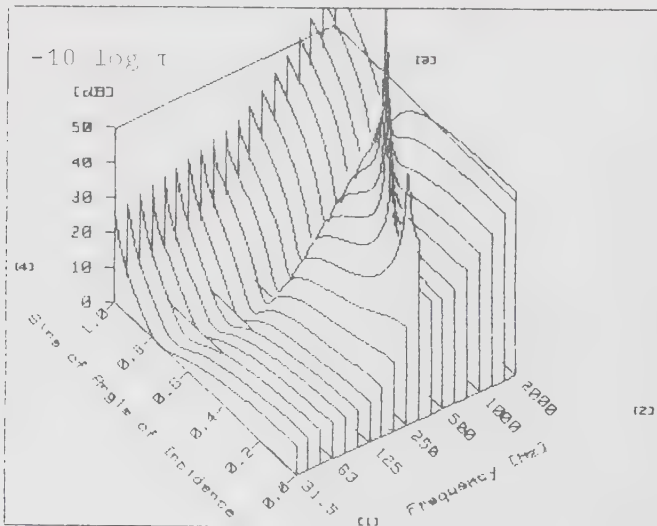
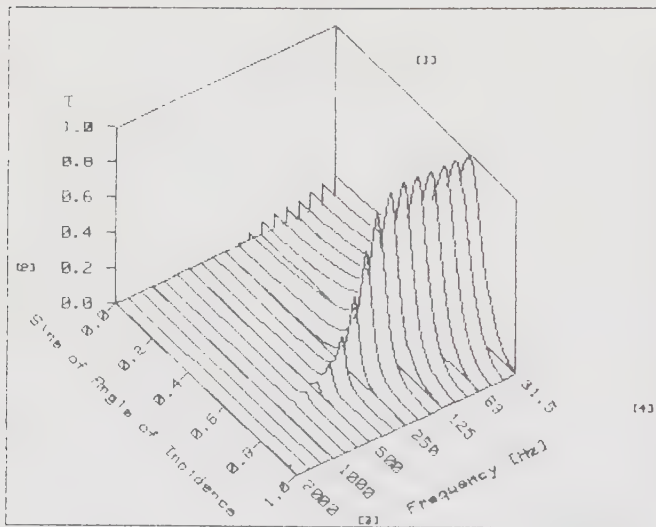
set $\theta = T = 0$, and choose a suitable value for m_0 . If we want to resemble the case of an excentric beam, we estimate θ and T for this beam and use these values in equations (2) and (3) together with a suitable m_0 .

This is just what we do in the example below. We take an excentric beam with $(h_0 \times b_0 = 2h \times 2h)$, which gives a total height $(h + h_0 = 3h)$. As in Part 1 we assume a slab thickness of 80 mm, and this gives us $\theta = 1.1 \text{ kg m}$, and estimating T in the previously discussed way gives $T = 2 \cdot 10^6 \text{ Nm}^2$. The extra mass of the beam itself is 59 kg/m, and to this we may add the load of a lightweight wall, which we estimated as 200 kg/m in Part 1. As we can see in the table below, we have assumed the dimensions of the symmetric beam to be $(b_0 \times h_0 = 0.224 \times 0.24 \text{ m})$ and this gives the correct θ , B_0 and h_{tot} for the resembled excentric beam. T and m_0 are calculated separately and used as input constants.

First we show the variation of τ and $-10 \log \tau$ with frequency and the angle of incidence:

MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	1.3E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m3]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	
Symmetric Blocking				
Mass	Bo x Ho [m]		0.224 x 0.240	
Density	Rho[kg/m3]		2300	
E-Modulus	E[Pa]		2.6E+10	
G-Modulus	G[Pa]		1.0E+10	
Extra Mass	Mo [kg/m]		259	
MassMom of Inert	[kgm]		1.1E+00	
Torsional Stiffn.	[Nm2]		2.0E+06	



Note the different viewpoint!

Here we have combined the cases from 2.3 and 2.4, and we would expect the result to have some resemblance to both the original cases.

The case of an elastic layer by itself had a simple $-10 \log \tau$ diagram, very low values at low frequencies and then steeply increasing for increasing frequency at all angles, except for grazing incidence where $-10 \log \tau = 0$.

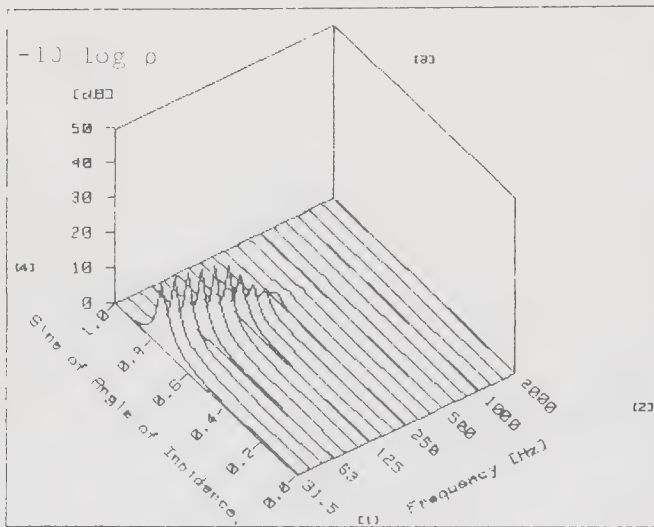
The case of the beam by itself had low values for $-10 \log \tau$ at low frequencies and small angles of incidence. At larger angles and higher frequencies the transmission loss increased, but it had two valleys of total transmission, and a ridge of total attenuation, besides the total attenuation at grazing incidence.

In our present case we have some resemblance to these two original cases. We have the lowest transmission loss at small angles of incidence and low frequency, and we also have increasing transmission loss for increasing frequency and larger angles of incidence. We may also note that we have a valley of total transmission at the same angle for all frequencies. This angle should correspond to $\sin \phi \approx \sqrt{h/h_0} = 0.58$ according to our discussions in part 2.4, but the phenomenon is seen to occur at about $\sin \phi = 0.7$ instead.

The other valley of total transmission, the angle dependant one, is not seen at all. In fact, instead of this valley, we have a ridge of total attenuation at about the corresponding angles and frequencies, where we would have expected this valley to be. In the one-dimensional case, this phenomenon of total attenuation was interpreted as the original f_A of the elastic layer by itself, shifted to a lower frequency by the influence of the extra mass. The angular and frequency variation here agrees with the predicted one for f_A in part 2.3.

Neither do we see any sign of the ridge of total attenuation, which we saw in the case of a reinforcing beam by itself.

At grazing incidence we have large attenuation at low frequencies, where the influence of the beam seems to dominate the transmission. At higher frequencies, where the elastic layer and the beam seem to influence the transmission more equally, we no longer have a peak indicating total attenuation, but instead a finite value at grazing incidence. For further orientation we may consider the figure for $-10 \log \rho$, shown below.

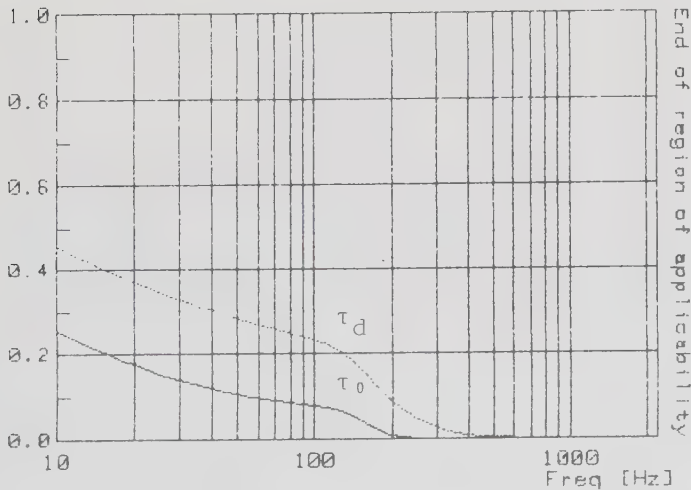


Also here we see only one ridge of total transmission, and it occurs at the same angle for all frequencies. We also see, that above about 250 Hz, where the transmission loss for the elastic layer itself starts to increase rapidly, this phenomenon of total transmission is weakened and disappears.

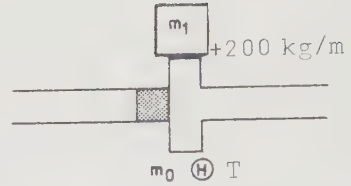
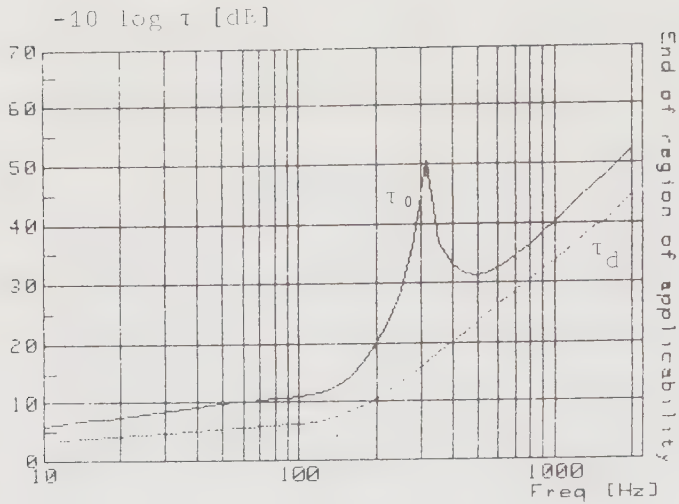
Our other phenomenon of total transmission from boundary condition i) has the same value of parameters here as in the example in 2.4, where it was not seen for frequencies below about 250 Hz, and this may explain why it is not seen at all in the figure above.

Finally we may point out that the ridge of total transmission from the case of the elastic layer by itself is not seen in the figure above either.

Let us finally consider the diffuse values and compare them to the ones for normal incidence. Below we show both the linear variation of τ with the frequency and the frequency variation of $-10 \log \tau$. The curves for $-10 \log \rho$ are not interesting here, as they have a value close to zero for all frequencies.



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at a Junction with an Elastic Layer
and a Symmetric Blocking Mass
Two Dimensional (Plates) [EBx2]



We see that the normal incidence case overestimates the improvement we get from a single sided reinforcing edge beam with an extra load. However, compared to $-10 \log \tau_d$ for the elastic layer by itself, we still have a significantly higher transmission loss, about 5 dB at low frequencies and about 10 - 12 dB at higher frequencies for our particular example.

We may also point out that the very high value for τ_0 , the top at f_A , is not seen in the curve for τ_d . It is totally levelled out when averaged over all angles of incidence.

2.5.2. Symmetric beams on both side of the elastic layer

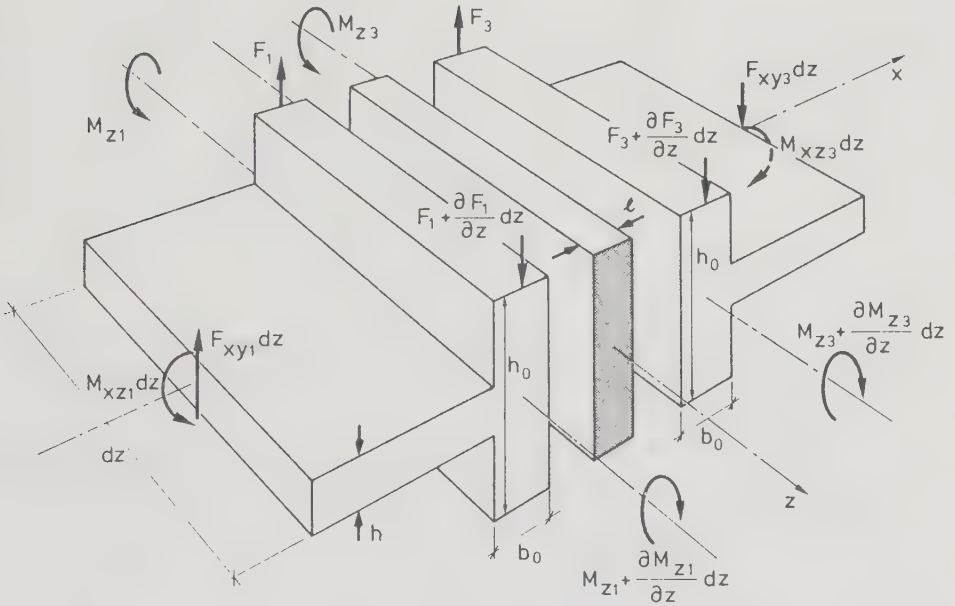


Figure 2.5.2. Configuration. As before we do not show v_y and w_z , but their positive directions are unchanged.

Finally we consider the case of reinforcing beams on each side of the elastic layer. As previously done we disregard the finite length of the elastic layer and of the beams, and get the following boundary conditions:

- i) $M_{xz1} - j\omega(\Theta - Tk_B^2 \sin^2 \phi / \omega^2) w_{z1} = M_{xz3} + j\omega(\Theta - Tk_B^2 \sin^2 \phi / \omega^2) w_{z3}$
- ii) $F_{xy1} - j\omega(m_0 - B_0 k_B^4 \sin^4 \phi / \omega^2) v_{y1} = F_{xy3} + j\omega(m_0 - B_0 k_B^4 \sin^4 \phi / \omega^2) v_{y3}$
- iii) $v_{y1} - v_{y3} = (j\omega / K') (F_{xy3} + j\omega(m_0 - B_0 k_B^4 \sin^4 \phi / \omega^2) v_{y3})$
- vi) $w_{z1} - w_{z3} = (j\omega / C') (M_{xz3} + j\omega(\Theta - Tk_B^2 \sin^2 \phi / \omega^2) w_{z3})$

We now write K' and C' to remind us of the larger contact area between the elastic interlayer and the concrete elements on each side.

Using the same help parameters as before, we can write the equation system as:

$$\begin{bmatrix} (-\psi_1 - j\alpha\psi) & (\sigma_1 - \alpha\sigma) & (\psi_1 + j\alpha\psi) & (-\sigma_1 + \alpha\sigma) \\ (-j\psi\sigma_1 + \delta) & (\sigma\psi_1 + \delta) & (-j\psi\sigma_1 - \delta) & (\sigma\psi_1 + \delta) \\ 1 & 1 & (-1 - \beta(j\psi\sigma_1 - \delta)) & (-1 + \beta(\sigma\psi_1 + \delta)) \\ j\psi & \sigma & (j\psi - \nu(\psi_1 + j\alpha\psi)) & (\sigma + \nu(\sigma_1 - \alpha\sigma)) \end{bmatrix} \begin{bmatrix} r \\ r_j \\ t \\ t_j \end{bmatrix} = \begin{bmatrix} \psi_1 - j\alpha\psi \\ -j\psi\sigma_1 - \delta \\ -1 \\ j\psi \end{bmatrix} \quad (4)$$

For normal incidence (4) becomes identical to equation (1.6.17) for the corresponding one-dimensional case.

The expressions for $r(\phi)$ and $t(\phi)$ are even more complex now, than in the case of a single sided beam, but for the sake of completeness we still write them out:

$$r(\phi) = \frac{R_r}{N_r + jN_i} \quad (5)$$

$$t(\phi) = \frac{jT_i}{N_r + jN_i} \quad (6)$$

where

$$\begin{aligned} N_r = & 4\nu(\sigma\psi_1^2) + 4\beta(\sigma\psi^2\sigma_1^2) + \nu\beta(\psi^2\sigma_1^4 - \sigma^2\psi_1^4) \\ & + \delta\{8\sigma + 8\nu + 2\beta(\psi^2\sigma_1^2 - \sigma^2\psi_1^2) - 4\nu\beta(\sigma\psi_1^2)\} + \delta^2(-4\beta\sigma - 4\nu\beta) \\ & + \alpha\{8(\sigma\psi^2) + 2\nu(\psi^2\sigma_1^2 - \sigma^2\psi_1^2) - 8\beta(\sigma^2\psi^2) - 4\nu\beta(\sigma\psi^2\sigma_1^2)\} \\ & + \alpha\delta\{4(\psi^2 - \sigma^2) - 8\nu\sigma - 8\beta(\sigma\psi^2) + 2\nu\beta(\sigma^2\psi_1^2 - \psi^2\sigma_1^2)\} \\ & + \alpha\delta^2\{2\beta(\sigma^2 - \psi^2) + 4\nu\beta\sigma\} \\ & + \alpha^2\{-4\nu(\sigma\psi^2) + 4\nu\beta(\sigma^2\psi^2) + \delta(2\nu(\sigma^2 - \psi^2) + 4\nu\beta(\sigma\psi^2))\} \\ & + \alpha^2\delta^2\{\nu\beta(\psi^2 - \sigma^2)\} \end{aligned}$$

$$\begin{aligned} N_i = & -16\sigma\psi - 4\nu(\psi\sigma_1^2) + 4\beta(\sigma^2\psi\psi_1^2) + 2\nu\beta(\sigma\psi\sigma_1^2\psi_1) \\ & + \delta\{-8\psi + 2\beta(\sigma\psi(4 + \sigma_1^2 + \psi_1^2)) + 4\nu\beta\psi\sigma_1^2\} + \delta^2(-4\nu\sigma - \psi) \\ & + \alpha\{8(\sigma^2\psi) + 2\nu(\sigma\psi(4 + \sigma_1^2 + \psi_1^2)) - 4\nu\beta(\sigma^2\psi\psi_1^2)\} \\ & + \alpha\delta\{5(\sigma\psi) + 8\nu\sigma\psi - 8\beta\sigma^2\psi - 2\nu\beta(\sigma\psi(4 + \sigma_1^2 + \psi_1^2))\} \\ & + \alpha\delta^2\{-4\beta(\sigma\psi) - 4\nu\beta\sigma\psi\} \\ & + \alpha^2\{-4\nu(\sigma^2\psi) + \delta(-4\nu(\sigma\psi) + 4\nu\beta(\sigma^2\psi) + 2\delta^2\nu\beta(\sigma\psi))\} \end{aligned}$$

$$\begin{aligned}
R_r = & -4\nu(\sigma\psi_1^2) + 4\beta(\sigma\psi^2\sigma_1^2) + \nu\beta(\sigma^2\psi_1^4 + \psi^2\sigma_1^4) \\
& + \delta\{-8\sigma - 8\nu + 2\beta(\sigma^2\psi_1^2 + \psi^2\sigma_1^2) + 4\nu\beta(\sigma\psi_1^2)\} + \delta^2(4\beta\cdot\sigma + 4\nu\beta) \\
& + \alpha\{8\sigma\psi^2 + 2\nu(\sigma^2\psi_1^2 + \psi^2\sigma_1^2) - 8\beta(\sigma^2\psi^2) - 4\nu\beta(\sigma\psi^2\sigma_1^2)\} \\
& + \alpha\delta\{8 + 8\nu\cdot\sigma - 8\beta(\sigma\psi^2) - 2\nu\beta(\sigma^2\psi_1^2 + \psi^2\sigma_1^2)\} + \alpha\delta^2(-4\beta - 4\nu\beta\cdot\sigma) \\
& + \alpha^2\{-4\nu(\sigma\psi^2) + 4\nu\beta(\sigma^2\psi^2) + \delta(-4\nu + 4\nu\beta(\sigma\psi^2)) + 2\delta^2\nu\beta\}
\end{aligned}$$

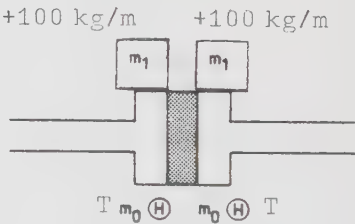
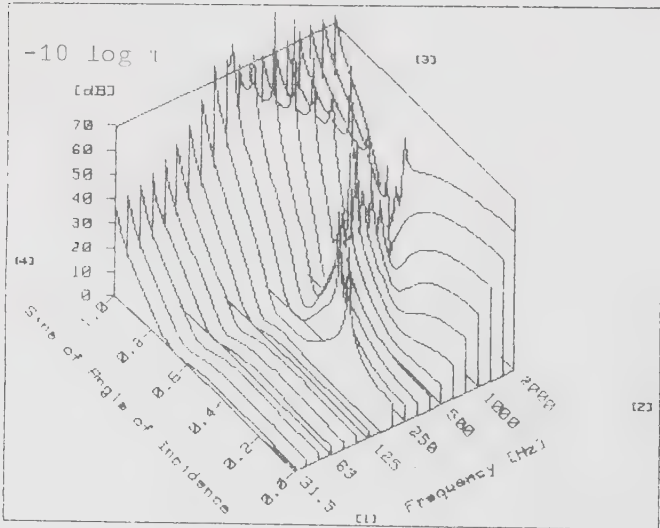
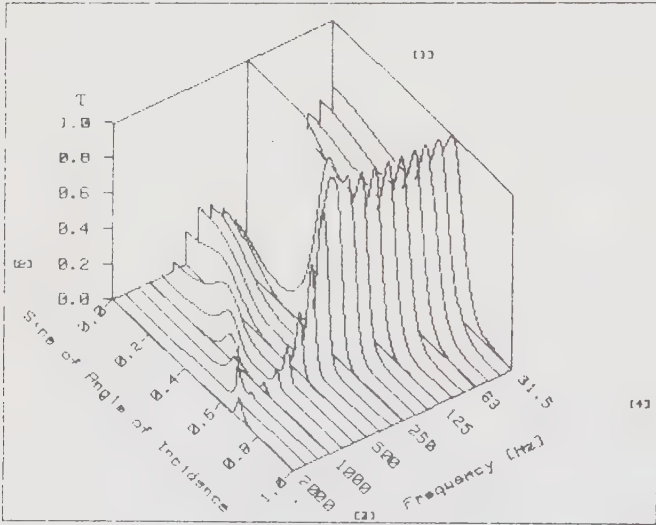
$$\begin{aligned}
T_i = & -16(\sigma\psi) - 4\nu(\psi\sigma_1^2) + 4\beta(\sigma^2\psi\psi_1^2) \\
& + \delta\{-8\cdot\psi + 8\beta(\sigma\psi\psi_1)\} + 4\delta^2\beta\cdot\psi \\
& + \alpha\{8\nu(\sigma\psi\sigma_1) + 8(\sigma^2\psi)\} - 4\alpha^2\nu(\sigma^2\psi)
\end{aligned}$$

As in part 2.5.1 we have programmed the analytical expressions (5) and (6), but the same comment applies here too, that if a standard routine is available for inverting the 4×4 matrix, it may be preferable to solve (4) numerically for each combination of angle and frequency.

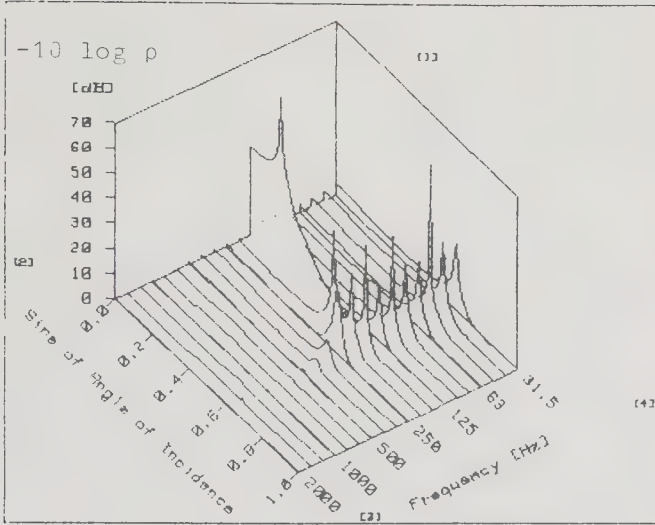
Here we show only one practical example, in which we have chosen the same beam as in part 2.5.1 on both sides of the elastic layer. Each beam is loaded with an extra mass of 100 kg/m. This same example was also used in the one-dimensional study. As in part 2.5.1 we use symmetric beams (0.224×0.24 m) which we assume to have the same θ , B_0 and h_{tot} as the practical ex-centric beams (with $b_0 \times h_0 = 0.16 \times 0.16$ m). T and m_0 are calculated separately and used as input constants.

First we show the angular and frequency variation of τ , $-10 \log \tau$ and of $-10 \log \rho$:

ELEMENT No:		1	2	3
L-modulus	E [Pa]	2.6E+10	1.3E+07	2.6E+10
G-modulus	G [Pa]		5.0E+06	
Density	Rho[kg/m3]	2300	800	2300
Poiss. ratio	My [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.240	0.080
Width of layer	L [m]		0.012	
Symmetric Blocking				
Masses	Bo x Ho [m]	0.224 x 0.240		0.224 x 0.240
Density	Rho[kg/m3]	2300		2300
E-Modulus	E[Pa]	2.6E+10		2.6E+10
G-Modulus	G[Pa]	1.0E+10		1.0E+10
Extra Mass	Mo [kg/m]	159		159
MassMom of Inert	[kgm]	1.1E+00		1.1E+00
Torsional Stiffn.	[Nm2]	2.0E+06		2.0E+06



Note the different viewpoint!



Note that the viewpoint has been changed again!

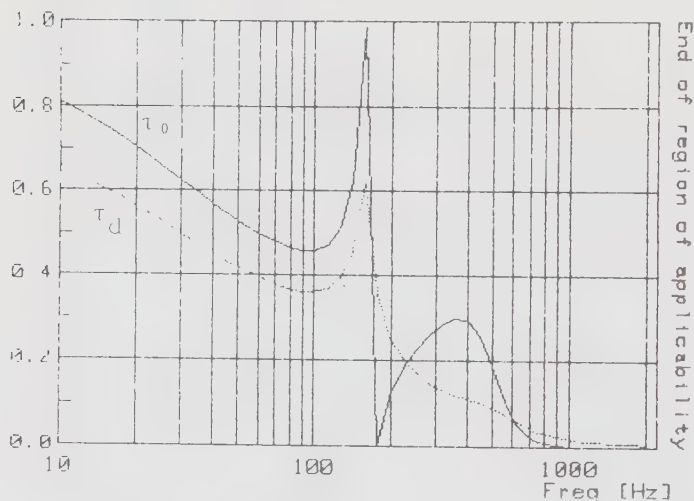
The general tendency in these figures is the same as, or similar to the case of the single sided beam in 2.5.1. We have low values for the transmission loss at low frequencies and for small angles of incidence. For an increasing frequency and for larger angles of incidence we have an increasing transmission loss in general, but there are many irregularities. The trace matching for the forces is important, and leaves a valley at about $\sin \phi = 0.65$. We can also see the trace matching phenomenon for the moments as a more shallow valley from about 250 Hz at normal incidence to about $\sin \phi = 0.7$ at 2000 Hz. This is more easily seen as a ridge in the τ -diagram, where we also can see the trace matching for the forces.

We can also see a third phenomenon of large transmission, giving $\tau \approx 1$ at about 160 Hz for normal incidence. The frequency dependence of this phenomenon is better seen in the diagram for $-10 \log \rho$, where we see that it occurs at an increasingly higher frequency

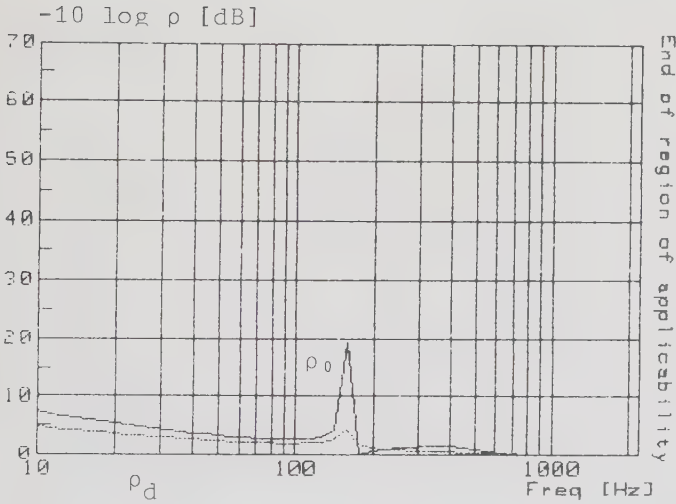
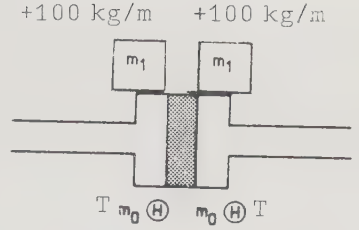
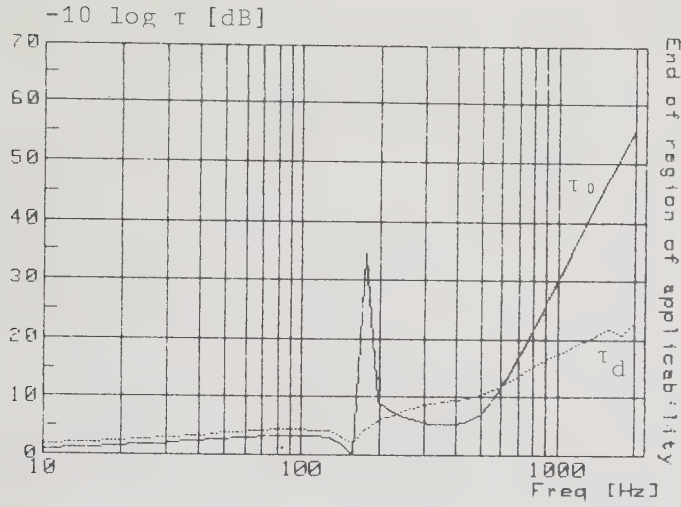
for an increasing angle of incidence. In the one-dimensional case we interpreted this f_T as the same phenomenon as for the elastic layer by itself, but that phenomenon has the opposite frequency variation with increasing ϕ , as we saw in part 2.3.

We also have several ridges of high attenuation in the diagram for $-10 \log \tau$. One is at grazing incidence, and another has started to develop into the ridge, which was seen to end up at f_A for a blocking mass at normal incidence in part 2.5.1. The third is the same phenomenon as for the elastic layer itself, giving f_A for normal incidence at about 160 Hz, and occurring at an increasing angle of incidence for an increasing frequency.

Finally we consider the diffuse values for τ , $-10 \log \tau$ and $-10 \log \rho$, and compare them to the ones for normal incidence as usual:



Transmission Efficiency for a Bending Wave
at a Junction with an Elastic Layer
and Symmetric Blocking Masses
Two Dimensional (Plates) [EBxx2]



First we may point out that the peak for $-10 \log \tau$ at f_A is smoothed out, and below that we have as in the case of normal incidence a very low transmission loss. Secondly the steep rise at high frequencies for τ_0 has turned into a more moderate rise, similar to the one we got for the elastic layer by itself. In fact if we compare the curve for $-10 \log \tau_d$ for the elastic layer by itself to the curve above, we find slightly higher values here (2 - 5 dB) up to about 1000 Hz, but at higher frequencies the curves are almost identical.

Finally we may point out that the peak for $-10 \log \rho$ at f_T is smoothed out in the diffuse case, and this is also seen in the $-10 \log \tau$ diagram, as the diffuse curve no longer goes down to zero there.

2.6 Summary and concluding remarks to Part 2

In part 2.3 we solved our fundamental case of an elastic layer between two identical slabs without any further constraints. The averaged result turned out to be very similar to the one for normal incidence. The slope towards higher frequencies is not as steep as for normal incidence, but the curve's shape is almost identical. The transmission loss is almost zero at low frequencies and then it increases by about 8 - 9 dB/octave. The high frequency asymptote intersects the frequency axis at about one 1/3-octave step above the f_T for normal incidence. The angular variation of f_T was found to be such that it moves to a lower frequency for a larger angle of incidence. The opposite was found for f_A .

In part 2.4 we presented and discussed Cremer's solution for a symmetric reinforcing beam on a plate. As a whole our 3-dimensional presentation agrees well with Cremer's interpretation of the involved phenomena. However, he has not observed that the phenomenon of total attenuation varies with frequency and the angle of incidence, and thus is not a purely normal incidence phenomenon.

The averaged result for the transmission loss turns out to be a curve with a similar form to the one for normal incidence, but the "blocking effect" at and above f_A is very much reduced. The phenomenon of total transmission is totally smoothed out, and instead of zero or almost zero transmission loss at low frequencies we have a value of a few dB. We investigated the influence of introducing a simplified form of corrected bending waves along the reinforcing beam, which has a much lower limiting frequency than the slabs for the applicability of simple bending wave theory. The influence of this correction was seen to

be quite marginal. The only noticable change was seen as a slight angular shift of the trace matching frequency for the forces. The averaged transmission loss was almost unaffected, but the maximum blocking effect shifted slightly, up to the same frequency as for normal incidence. As the total effect of this correction is so marginal, it is disregarded in part 2.5.

Finally in part 2.4, we discuss the fact that the practical beams in our case are excentric and not symmetric as in the calculations. We state that the error because of this may be a little larger here than in the one-dimensional case, but still the calculations will give a fair estimate of the bending wave transmission loss. As in Part 1 the involved parameters are calculated for the real excentric beam, and then used in the theoretical model for the symmetric one.

In part 2.5 we combine the elastic layer and boundary constraints from reinforcing edge beams and extra mass loads. First we consider the case of single sided boundary constraints, corresponding to the practical situation of the elastic layer on one side of a lightweight wall on a reinforcing beam. Secondly we consider the elastic layer with double sided boundary constraints. This corresponds to the more usual case of the elastic layer separating two reinforcing beams, each carrying the load of half the lightweight wall.

The result is that the single sided case gives higher averaged transmission loss than the double sided one, although the difference is not as large as for normal incidence. The observed reduction in transmission loss by introducing the second edge beam is partly because the symmetric case has a strong phenomenon of total transmission which does not occur for the single sided case. However, an even more important factor is

the larger contact area between the elastic layer and the concrete elements on each side.

Now the practical realization of the single sided case may be somewhat problematic. We may also point out that even in the single sided case, we still have a large transmission in the low frequency region of building acoustics. Thus this measure would not in itself solve the problem for our practical example, we would also have to use other means, such as choosing a softer interlayer (less shear stiffness).

The detailed appearance of the 3-dimensional figures for both cases is seen to be in accordance with our expectations, except perhaps for some minor deviations. For the more detailed discussion of the angular and frequency variation we refer to the text in part 2.5.

PART 3

EXPERIMENTAL STUDY

3.1 Introduction

In the third part of this report, we first present a systematic measurement study of the sound transmission between concrete slabs on an elastic foundation, coupled in different ways. The measurements are performed in a laboratory, which means that the size of the slabs has been reduced, and the boundary conditions are not exactly the same as for the full scale cases.

Secondly we publish the measurement results for the properties of the materials we tested as interlayers. These material properties are then used to make a few comparisons between our theory and the measured values for the transmission loss.

Finally we publish the field measurements that we have made and we also discuss different ways of increasing the losses in the slabs. This would be the safest way of reducing the flanking transmission through the concrete slabs.

3.2 The transmission loss measurements

3.2.1 Equations for calculations of the measured transmission loss between plates

Similar to the way in which transmission loss for the airborne sound insulation of a wall is measured, we may measure the transmission loss for a junction between two plates. We assume a diffuse field of bending waves in each plate, and measure the difference in vibrational level between them. Similarly to the airborne case, we have to know the losses of the receiving system and the length of the junction to obtain a normalized value of the transmission loss. The plates have inner losses and they also have losses because of power transmission to other structural elements to which they are coupled. These losses are usually referred to as "edge absorption". In our case we have very little edge absorption, mainly inner losses and losses to the elastic foundation. Both these phenomena are thus related to the area rather than the edges of the plate, but we may still convert the losses into an equivalent edge absorption Λ . (Similarly to an equivalent absorption area, A , in a room.)

If we denote the length of the junction as L , we can now write

$$R = -10 \log \tau = L_s - L_m - 10 \log (\Lambda_m/L) \quad (1)$$

where the index s denotes the source plate and the index m denotes the receiving plate.

The equivalent edge absorption can be calculated by using the measured reverberation time (T_{60}) for the slab or the measured loss factor ($\eta = 2.2/(\tau_{60} \cdot f)$):

$$\Lambda = \frac{13.8 \cdot \pi \cdot S}{2c_B \cdot T_{60}} \quad (2)$$

$$= \frac{\omega \eta \pi S}{2c_B} \quad (3)$$

(See e.g. [14].)

In the equations above, we have assumed that the group velocity of bending waves in the plates can be written as $c_{Bg} = 2c_B$. However, as the plates are resting on an elastic foundation we may have to consider the stiffness (K) of the foundation in the bending wave equation:

$$B \nabla^4 v - \omega^2 m v + K v = 0 \quad (4)$$

This gives us the bending wave number

$$k_B = \sqrt[4]{(\omega^2 m - K)/B} \quad (5)$$

$$= \frac{\omega}{c_B} \sqrt[4]{1 - (\omega_0/\omega)^2} \quad (6)$$

where $\omega_0^2 = K/m$

$$c_B = \sqrt{\omega} \sqrt[4]{B/m}$$

As the group velocity is $c_{Bg} = \frac{d\omega}{dk_B}$ we have:

$$c_{Bg} = 2c_B (1 - (\omega_0/\omega)^2)^{3/4} \quad (7)$$

For $\omega \gg \omega_0$ we see that (7) reduces to $c_{Bg} = 2c_B$.

3.2.2 The measurement objects

The measurement objects were two equally large concrete slabs, 3.00 x 2.20 x 0.08 m each. The shorter edge length was used as the transmitting one.

Two types of elastic foundations were tested:

- (A) 80 mm Styrolit 35221080, 30 kg/m³ (expanded plastic)
- (B) 70 mm Rockwool 389-00, 150 kg/m³.

As elastic interlayers we tested 13 mm asphalt impregnated porous fibre board, both in single and double layers. (This material is called "asfaboard" for the rest of the report.) We also tested both types of the foundations above as an interlayer as well. Finally a special type of interlayer was tried, where the asfaboard was sawed into strips, which were stacked on top of one another.

In practice the slabs are cast with the interlayer as a part of the casing, so the contact is very good between the interlayer and the slabs. To achieve a good contact in the laboratory measurements, the slabs were pressed together with a 6 kN force before a measurement was made, and then the force was reduced to only 0.5 kN during the measurement. Separate edge beams were cast to use in some of the measurements. However, their fastening to the slabs and the contact with the interlayer did not give a satisfactory resemblance to the real case of the edge beams and the slabs cast in one piece. The situation is more similar to the case of a lightweight wall at the junction.

3.2.3 Acceleration level measurements

As it is only the difference in level between the two slabs that we are interested in, it does not matter if the level in question is the acceleration level, velocity level or displacement level. We confined ourselves to measurements of the acceleration level. The source

slab was excited in one of its corners with an electrodynamic shaker, fed with pink noise. Eight accelerometer positions were used on each slab, and the average value calculated. The accelerometer positions were permanently glued small metal plates, on which the accelerometer was fastened with a screw.

3.2.4 Measurements of the loss factor

The loss factor of the slabs was measured by exciting the slabs with a hammer blow and registering the plate resonances (modes) with an accelerometer coupled to an FFT-analyzer. The resonance frequency (f_N) was measured together with the half-value bandwidth (b) and the loss factor calculated by:

$$\eta(f_N) = b/f_N \quad (8)$$

We also tried exciting the slabs with the shaker fed with PN-noise from the FFT-analyzer. This was seen to give identical results, but the hammer method was preferred, as it is convenient to separate double resonance peaks by simply knocking at a slightly different place on the slab.

Several resonance peaks were measured for each of the two types of foundations. The results were plotted on a log-log diagram, where they seemed to form approximately a straight line. We calculated the mean square line for each of the two cases, and from this line we determined the values of η at each 1/3 octave frequency.

The results are shown in Appendix III.1 and III.2.

3.2.5 Presentation of the transmission loss measurements

In Appendix I we present the different transmission loss measurements. In every diagram we show the level difference (ΔL) between the plates, and also the normalized transmission loss (R) according to equations (3.2.1) and (3.2.3). (The influence of the foundation on the bending wave group velocity is disregarded.) The values of R are also shown in a table.

The measurements where the expanded plastic is used as a foundation are denoted "A", and the ones where the mineral wool is used are denoted "B".

Our first measurement series is made on the foundation "A". We test simple and double asfaboard and cellular plastic as interlayers, and for comparison we make measurements for an airgap and for direct contact between the slabs. To find out if any transmission takes place via a continuous foundation of cellular plastic, we measure this configuration with an airgap and with a single asfaboard. Finally we make a measurement for a single asfaboard with the separate edge beams mounted, and we also make one measurement without the compression, to see if this changes the result.

The order of these measurements in Appendix I is as follows:

	INTERLAYER	FOUNDATION	"ID"
I.1	Single asfaboard (13 mm)	A	A2
.2	Double asfaboard (2x13 mm)	A	A4
.3	Cellular plastic (80 mm)	A	A25
.4	Airgap (13 mm)	A	A3
.5	Slabs in direct contact	A	A26
.6	Single asfaboard (13 mm)	A,continuous	A17
.7	Airgap (13 mm)	A,continuous	A24
.8	Single asfaboard (13 mm) (Extra load, 150 kg on each side of the junction.)	A	A16
.9	Double asfaboard (2x13 mm) (No compression during the measurement.)	A	A5

Our second measurement series is made on foundation "B". At first we test different types of interlayers: single and double asfboard, cellular plastic, 70 mm mineral wool, the special "stripped" asfboard (26 mm and 52 mm) and finally an airgap. Next we investigate the effect of doubling and redoubling the width of an interlayer of mineral wool. We chose mineral wool with the same density as foundation "B", but with a width of only 50 mm. Finally we investigate the effect of extra mass loading at one side or both sides of the junction. As interlayers we used the 50 mm mineral wool and the single asfboard.

The order of the measurements in this second series in Appendix I is as follows:

INTERLAYER		FOUNDATION	EXTRA LOAD	"ID"
I.10	Single asfboard (13 mm)	B		B14
.11	Double asfboard (2x13 mm)	B		B15
.12	Cellular plastic (80 mm)	B		B18
.13	Mineral wool (70 mm)	B		B12
.14	Stripped asfboard (26 mm)	B		B16
.15	Stripped asfboard (26 mm)	B		B17
.16	Airgap (13 mm)	B		B10
.17	Mineral wool (50 mm)	B		B36
.18	Mineral wool (2x50 mm)	B		B35
.19	Mineral wool (4x50 mm)	B		B33
.20	Mineral wool (50 mm)	B	1x150kg/m	B37
.21	Mineral wool (50 mm)	B	1x430kg/m	B38
.22	Mineral wool (50 mm)	B	2x280kg/m	B43
.23	Asfboard (13 mm)	B	1x430kg/m	B39
.24	Asfboard (13 mm)	B	2x280kg/m	B44

3.2.6 Comparison of the results in pairs

In Appendix II we compare in pairs several of the curves from Appendix I. We try to do this systematically and discuss one parameter at a time.

First we consider the different types of foundation, and these curves are presented in Appendix II.1 The

effect of having the foundation of cellular plastic continuous is studied in II.1.1 and II.1.2. In the case of an airgap between the slabs, no significant difference is seen between the cases of continuous or interrupted foundations. However, in the case of an asfaboard interlayer we seem to have about 5 dB lower transmission loss for the continuous case. This discrepancy between the two cases is difficult to explain.

In II.1.3 - II.1.5 we compare the foundations of type A and B. First we may point out that if we have a junction with a high transmission loss, such as the airgap, we have higher flanking transmission at low frequencies for foundation A than for foundation B (see II.1.3). However, this flanking transmission does not affect the transmission loss measurements, as the curves for the different interlayers lie well below the "maximum curve" for the airgap. (The only exception is the 200 mm interlayer of mineral wool (B33), which has values of the same order of magnitude as the airgap at high frequencies.) Anyway, considering the curves in II.1.2 - II.1.3, we can state that the type of interlayer plays a minor role for the resulting transmission loss.

In Appendix II.2, we study the asfaboard as an interlayer. In II.2.1 a single asfaboard is compared to the case of a direct contact between the slabs. We see that the asfaboard does not improve the transmission loss, except for frequencies above approximately 600 - 800 Hz, where the improvement is about 10 dB. In II.2.2 we compare the asfaboard and the airgap, and we see that the difference is about 20 dB, so we can feel sure that the measurement is unaffected by the flanking transmission. In II.2.3 - II.2.4 we compare a single and a double asfaboard, and the difference is shown to

be only marginal. In II.2.4 the double board gives a slightly better result, but in II.2.3 the single board is slightly better.

In Appendix II.3 we study the two types of foundations as interlayers. In II.3.1 - II.3.2 we compare A and B to the case of the airgap, and we see that we have a difference exceeding 10 dB at (almost) all frequencies, so the influence of the flanking transmission should be only marginal. In II.3.3 we compare the two interlayers A and B. The difference curve is rather irregular, A has the highest transmission loss at the lower frequencies, and B at the higher frequencies. As a whole we may judge the two to be just about equally good as insulating interlayers. Finally we may compare the asfaboard to one of A and B, and in II.3.4 we do this for B. We see that by using B (or A) instead of the asfaboard, the transmission loss is increased by roughly 10 - 15 dB in almost the entire frequency region.

In Appendix II.4 we consider the stripped asfaboard as an interlayer. The asfaboard strips should have much less shear stiffness than the original asfaboard, and thus a considerably higher transmission loss. Two widths of the strips were tested, 26 mm and 52 mm. (Note that 26 mm is the same width as the double original board.) We start in II.4.1 by comparing the 52 mm strips to the airgap. We see that the difference is larger than 10 dB for all frequencies. In II.4.2 we compare the two different strip widths. At low frequencies the wider layer has about 5 dB higher transmission loss, but at higher frequencies the narrower one has about 2 - 3 dB higher value. Compared to the cellular plastic, A, as an interlayer, the 52 mm strips almost have an identical transmission loss, as seen in II.4.3. However, the low values for A at about 300 Hz

are not experienced in the case of the stripped asfaboard, which gives it a few dB advantage there.

Finally in II.4.4 we compare the 52 mm strips to the single asfaboard. The strips have a much higher transmission loss as expected, up to 20 dB at intermediate frequencies. As the result for the double asfaboard was similar to the one for the single asfaboard, and the same was true for the 52 mm and the 26 mm strips we can also state that 2 x 13 mm of the original asfaboard has a much lower transmission loss than the 26 mm asfaboard stripes. The boards are pressed in such a way that the fibres are mostly orientated in a direction parallel to the surface. By dividing the boards into strips and stacking them in the joint, we would expect the longitudinal stiffness to be increased, while the shear stiffness is decreased, compared to the case of the original boards.

In Appendix II.5 we consider different widths of mineral wool as an interlayer. In II.5.1 we compare 50 mm and 2 x 50 mm layers. The difference is small at low frequencies and is roughly 5 dB above 600 - 800 Hz. In II.5.2 we compare 50 mm and 4 x 50 mm layers. Here we have a difference of about 5 dB at low frequencies, and about 10 - 15 dB at high frequencies. Finally we also show the comparison between the cases of 2 x 50 mm and 4 x 50 mm layers. We may point out, that the case of 4 x 50 mm layer reaches about the same values for the transmission loss at high frequencies as the airgap (B10). (This comparison is not shown separately.) The values at frequencies above approximately 1000 Hz are thus probably reduced because of flanking transmission.

Our final comparison series considers the influence of extra constraints at the junction in the form of extra

masses. In II.6.1 - II.6.3 we have the asfboard as an interlayer and in II.6.4 - II.6.6 we have the 50 mm mineral wool as an interlayer. In II.6.1 we try a rather moderate load (150 kg/m) on each side of the asfboard interlayer, which does not give any significant change. In II.6.2 we have increased the load on each side to 280 kg/m, and here we obtain an increased transmission loss of about 8 - 10 dB over most of the frequency region. In II.6.3 we have a large load on only one side of the junction, but no significant improvement is achieved, which is rather surprising, considering our theoretical study. In II.6.4 a moderate single sided mass is added to the configuration of a 50 mm interlayer of mineral wool. Also here the result is rather surprising, we seem to have introduced a phenomenon of high transmission at about 160 Hz. At intermediate frequencies we have obtained a slight improvement, but at high frequencies we seem to have another phenomenon of high transmission. Similar tendencies can be seen for the larger single sided extra mass in II.6.5, but the valleys of high transmission are not as deep here. This behaviour does not seem to be in accordance to our theory in Part 1 and Part 2. Finally we see in II.6.6, that a relatively large load on each side of the junction gives an improved transmission loss of roughly 5 - 10 dB, similar to the corresponding case of the asfboard.

Our very last case is shown in II.7.1, where we investigate if the 0.5 kN compression during all the measurements has any influence on the results. No measurable difference is found between the cases of compression or no compression.

3.3 Measurements of the properties of the test materials

3.3.1 Method

The different materials that we tested, were:

1. Two types of asfboard, 12-13 mm and 14 mm
2. Asfboard strips 26 and 52 mm
3. Cellular plastic (30 kg/m^3) 80 mm
4. Mineral wool (150 kg/m^3) 70 mm and 50 mm

The material properties to be measured were primarily the dynamic modulus of elasticity (E) and the dynamic shear modulus (G). We also measured the density (ρ), and the loss factor was determined from the -3 dB value of the resonance peaks.

The measurements were made by using samples of the test materials as spring elements in a mass-spring system. The mass was excited with a hammer blow and its acceleration was measured with the help of accelerometers and an FFT-analyzer.

Several types of masses were tried, and as we did not use any glue to fasten the test specimen to the mass or to the foundation, we found out that the mass had to be rather heavy. This was especially the case for the asfboard and the cellular plastic, which did not have a sufficient contact with the foundation, when relatively light masses were used.

The more heavy masses that we finally used, were a steel cube $0.242 \times 0.242 \times 0.153 \text{ m}^3$ (70.0 kg) and also concrete tiles $0.35 \times 0.35 \times 0.05 \text{ m}^2$ (14.5 kg), which we stacked on top of one another. The accelerometers were fastened with bees wax.

When we measured the E-modulus, two accelerometers were fastened on the top of the mass and the mass was excited with a hammer blow from above (see the figure below.)

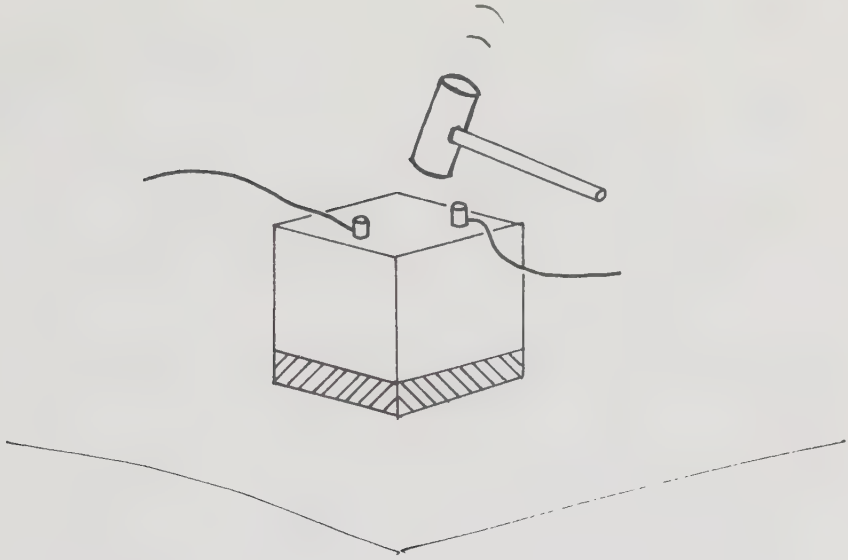


Figure 3.3.1. Measurement of the E-modulus.

When the G-modulus was measured, the accelerometers were placed in a similar way on one side of the mass, and the mass was then excited with a blow on the opposite side.

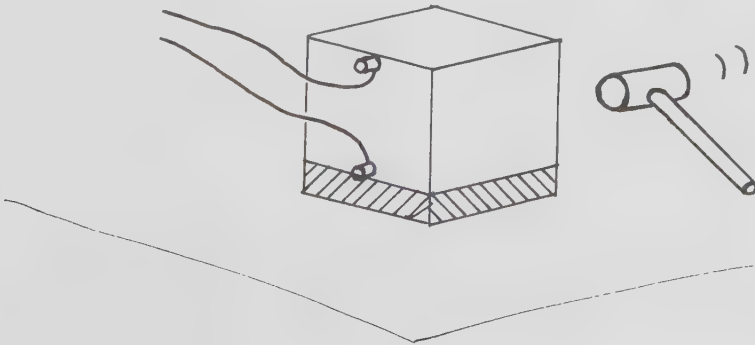


Figure 3.3.2. Measurement of the G-modulus.

The signal from the accelerometers was analyzed in a two-channel FFT-analyzer. A typical result is shown in Appendix IV.1, where the upper diagram shows the amplitude spectra for the signal from the two accelerometers. As we can see, it is difficult to interpret what kind of resonances the two peaks represent. The lower diagram shows the peak spectra, and for this particular example we see that the accelerometers are "in phase" at the first resonance, but have a 180° phase difference at the second one. Thus the first one is the SDOF mass-spring resonance that we are searching for, while the second one represents a see-saw movement of the mass.

When the resonance frequencies f_G and f_E have been found, the modules can be calculated by:

$$G = (2\pi f_G)^2 \cdot \frac{M}{S} \cdot l \quad (1)$$

$$E = (2\pi f_E)^2 \cdot \frac{M}{S} \cdot l \quad (2)$$

where M = the mass [kg]

S = the contact area between the mass and the test specimen [m^2]

l = the width of the test specimen [m]

The "-3 dB bandwidth" (b) was also measured, and the loss factor calculated by:

$$\eta = b/f_E \quad (3)$$

3.3.2 Results for the asfaboard

Two types of asfaboard were tested, and the results were almost identical for both types. Primarily we measured the 12-13 mm (black) asfaboard which was

used as an interlayer in the transmission loss measurements. Secondly we measured another type of 14 mm (brown) asfboard which is used in the test of resonance damping of the slabs (see part 3.4.2).

In the table below we show the average values of several measurements.

Table 3.3.2.

	12-13 mm (black) asfboard	14 mm (brown) asfboard
ρ (kg/m ³)	265	285
E (Pa)	$3 \cdot 10^6$	$4 \cdot 10^6$
G (Pa)	$2 \cdot 10^7$	$2 \cdot 10^7$
η	0.2	0.2

It may at first seem surprising that the G-modulus is larger than the E-modulus, but as we explained previously the fibres in the boards are orientated in a direction parallel to the surface. This gives an anisotropic material with a much higher stiffness in the plane of the boards than perpendicular to them. The large difference in table 3.3.2 may still be somewhat surprising. (Of course the relation $G = E/(2(1+\mu))$ is not valid for anisotropic materials.)

We may finally point out that our result differs from the previously cited one in [30], which we have used as an example in our theoretical considerations in Part 1 and Part 2. (In [30] the results were $G = 4.9 \cdot 10^6$, $E = 1.8 \cdot 10^7$ Pa.)

3.3.3 Stripped asfaboard

One type of interlayer consists of strips of the black asfaboard, 26 mm and 52 mm wide. We made attempts to measure the modules for these two types. The strips were stacked as shown in the figure below, then they were pressed together and the mass placed on top of them. The modules were then measured in the indicated directions.

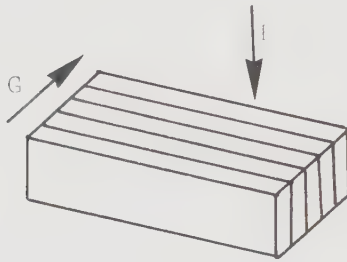


Fig 3.3.3. Measurement of stripped asfaboard.

Several measurements were made, and the average results are shown in the table below:

Table 3.3.3.

	26 mm strips	52 mm strips
ρ (kg/m ³)	265	265
E (Pa)	$2 \cdot 10^7$	$6 \cdot 10^7$
G (Pa)	$2 \cdot 10^6$	$2 \cdot 10^6$
η	0.1	0.1

If we compare the modules for the strips to the ones for the original boards, we find that the G-modulus

has decreased by a factor 10, while the E-modulus has increased by a factor 10 - 30. (The difference in E-modulus between the two widths of strips is of course rather surprising.)

3.3.4 Results for the cellular plastic

The 80 mm cellular plastic was used both as an inter-layer and as a foundation.

Several measurements were made with a rather large spreading of the results for the G-modulus. No systematic variation of the E-modulus with different static loads was registered. In the table below we show the average measurement results.

Table 3.3.4.

	80 mm cellular plastic
ρ (kg/m ³)	30.0
E (Pa)	$9 \cdot 10^6$
G (Pa)	$2 \cdot 10^6$
η	0.1

This material should be approximately isotropic so the large difference between E and G is unexpected. We had considerable spreading of the results for G, but hardly enough to account for this large difference. There may be some anisotropy in the material after all.

3.3.5 Results for the mineral wool

Dense mineral wool (150 kg/m^3) was used both as an interlayer and as a foundation. Two different widths were used, 50 mm and 70 mm.

The measured modules showed a very strong dependence on the static load. The E-modules were measured for loads up to 20 kPa, and the results are shown in Appendix IV.2.1 - IV.2.2. We also measured the static deflection and calculated the static modules as well. The G-modulus was difficult to measure for large static loads (high stacks of concrete tiles!), but it too showed clear tendencies in the direction of higher values for increased static loads.

In the table below we show the values for E, G and η for two different static loads, 1.2 and 2.4 kPa. For values of E at larger static loads we refer to Appendix IV.2.1 - IV.2.2. No reliable measurements were obtained for G at larger static loads, but η was measured as a function of the static load for the 50 mm mineral wool, and the result is presented in Appendix IV.2.3.

Table 3.3.5.

Statical load	70 mm Rockwool (150 kg/m^3)		50 mm Rockwool (150 kg/m^3)	
	1.2 kPa	2.4 kPa	1.2 kPa	2.4 kPa
E (Pa)	$4.5 \cdot 10^5$	$5.4 \cdot 10^5$	$3.8 \cdot 10^5$	$5.0 \cdot 10^5$
G (Pa)	$7 \cdot 10^5$	$8 \cdot 10^5$	$8 \cdot 10^5$	$1.3 \cdot 10^6$
η	0.3	0.2	0.34	0.27
$\rho \text{ (kg/m}^3\text{)}$	165		160	

Just as in the case of the asfaboard we have here a higher G-modulus than E-modulus. The mineral wool is very anisotropic as well as the asfaboard. The fibres are orientated in a direction parallel to the surface, giving a much higher modulus in this direction than the perpendicular one.

We made strips of the 50 mm mineral wool and measured the E-modulus in the direction perpendicular to the previous one. Only one measurement was made, and the result was $E = 1.3 \cdot 10^7$ Pa for a static load of 12 kPa. This is about 15 times higher than the corresponding value in Appendix IV.2.2.

3.4 Special considerations

3.4.1 Field measurements

A few field measurements of the transmission loss of junctions with an elastic interlayer have been made. In Appendix I.25 - I.27 we show three such measurements. The first one has cellular plastic as an interlayer, The two other ones have an airgap, as three boards were used as a casing when the slabs were cast and then the middle one was removed.

The case with the cellular plastic as an interlayer has unexpectedly low values for the transmission loss, compared to the closest corresponding laboratory measurement. The two cases are compared in Appendix II.8.1. An inspection at the building site revealed a mechanical contact between the two slabs, which probably explains the large difference between the two cases.

The other two cases are not directly comparable to any one of our laboratory cases, but we may state that this solution gives a relatively high transmission loss, similar values to the values for the interlayers A and B in our laboratory measurements.

In II.8.2 we compare the two cases from I.26 and I.27. The only difference between the two is the continuous or interrupted foundation of cellular plastic respectively. We see that we do indeed have some transmission via the continuous one, approximately 5 dB at the lower frequencies. This is a similar result to the one we obtained in II.1.2.

The loss factor measurements for these field measurements gave similar results to our laboratory measure-

ments for the slabs on mineral wool, and are not shown separately.

However, for comparison we wanted to measure the loss factor and the transmission loss for a case, where the concrete slab was in direct contact with the ground. No sound insulation problems because of flanking transmission in the concrete floors have ever been experienced for such cases. We found such a measurement object, but the losses in the floor were so high, that no resonant bending wave field was formed. Instead the acceleration level decreased successively for measurement positions further away from the shaker. The same was true in the receiving plate, the level decreased successively as we measured further away from the junction. Thus we were not able to make any transmission loss measurements, but in Appendix III.3 we show the result of the loss factor measurement.

The foundation was made of expanded clay (Leca) and we see that the slabs have a loss factor of the order of magnitude 10 times larger than for the case of floating slabs on mineral wool.

The large loss factor does of course not improve the transmission loss for the junction, but it reduces the level in both slabs (assuming constant excitation). The flanking transmission is thus reduced.

3.4.2 Increasing the loss factor through a resonance method

We have now seen, that the transmission loss can be increased by using interlayers with less shear stiffness. However, even if we use interlayers with an extremely low shear stiffness, there is always a risk of

a short circuit by an accidental mechanical contact. As the acoustic temperature of the slabs is very high, this may lead to a large flanking transmission.

A solution that is not so sensitive to accidental mechanical contact would be preferable. This solution would be to lower the acoustic temperature by increasing the losses. If we insist on having the slabs floating on an elastic foundation we would have to increase the inner losses of the slabs. This could be done by using a thin interlayer of a viscoelastic material between two slabs on top of each another. However, this method is rather expensive, and would probably be rejected because of that.

The flanking transmission is mainly a problem at frequencies in the region 100 - 500 Hz, so a selective damping through resonance methods may be an alternative. We have tried one such arrangement where we introduced an additional mass below the 80 mm slab with a resilient layer in between. This gives us a system of two springs and two masses:

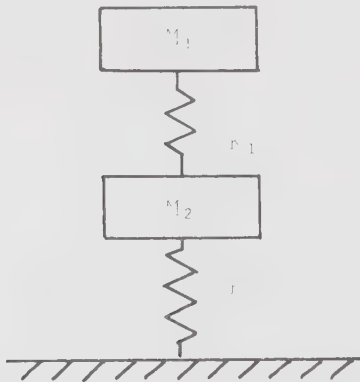


Fig 3.4.2. Scematic picture of the resonance system.

This system has the resonance frequencies:

$$\omega_0^2 = \frac{K_1}{2M_1} + \frac{K_2+K_1}{2M_2} \pm \frac{1}{2} \sqrt{\left(\frac{K_1}{M_1} + \frac{K_1+K_2}{M_2}\right)^2 - \frac{4K_1K_2}{M_1M_2}} \quad (1)$$

In our case M_1 represents the 80 mm concrete slab and K_2 the 70 mm mineral wool layer. As M_2 we introduced 50 mm concrete tiles, and as K_1 a 14 mm (brown) asfa-board.

According to the measured values in 3.3, we would expect resonances at about 315 Hz and at about 25 Hz.

In Appendix III.4 we show the results from the loss factor measurements for this configuration. For comparison we show the values for the slab on just mineral wool. We see that we have managed to increase the value of the loss factor approximately 4 - 6 times at the lowest frequencies.

In III.5 we have averaged the values within each 1/3-octave band. We have the highest value at about 250 Hz, $\eta \approx 0.14$, which we may compare to the value $\eta \approx 0.24$ for the slab on expanded clay and $\eta \approx 0.02$ for the slab on mineral wool only.

Finally in Appendix II.9.1 we show the difference in measured acceleration level of the receiving slab when it rests on mineral wool only or the above described damping system respectively. We see that we have a difference of about 5 dB at the lowest frequencies. This may not seem much, but if the method is applied to the source slab as well, we would expect the double effect, assuming that the excitation is unchanged.

Still, this may be considered to be a relatively complex and expensive method, and the result is perhaps

not so good, that it will be considered as a practical alternative.

3.5 Comparison between theory and measurements

Finally we would like to compare some of our measured results to our theory from Part 2. In part 3.3 we presented measurement results on the properties of the different materials used as interlayers and foundations. These values will be used in the calculated results below.

First we may point out that according to the measured value of the dynamic modulus of the cellular plastic, we have a fundamental resonance at approximately 125 Hz. This means that we would have to use the corrected bending wave velocity according to equation 3.2.7 when calculating the transmission loss for the cases which have the cellular plastic as a foundation.

This has, however, not been done. Our model of diffuse bending wave fields in the slabs becomes unapplicable at the lowest measured frequencies. At 160 Hz the corrected group velocity has only half the value $2c_B$ and the resulting correction of the measured transmission loss is therefore -3 dB, and decreases with increasing frequency.

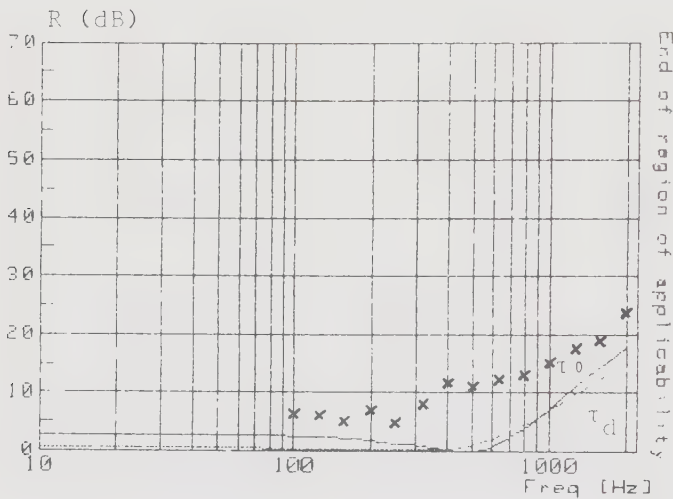
The mineral wool foundation on the other hand gives a fundamental resonance at approximately 30 Hz so no corrections of this kind have to be made.

In the following considerations we will confine ourselves to cases of the mineral wool foundation.

Let us as a first example take the single asfaboard as an interlayer. This gives the following result using material data from table 3.3.2:

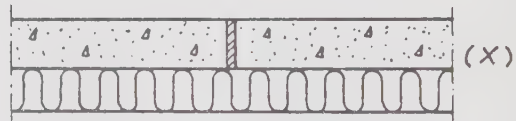
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	3.0E+06	2.6E+10
G-modulus	G [Pa]		2.0E+07	
Density	Rho[kg/m ³]	2300	265	2300
Poiss. ratio	My [1]	0.25		0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	



B 14

Foundation: Mineral wool (70 mm)
 Interlayer: Single asfaboard (13 mm)

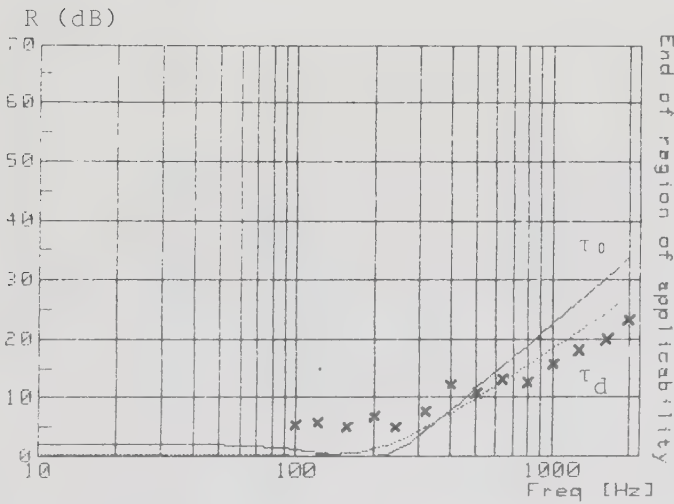


As we see, the agreement between theory and measurement is not very good. If we assume the theory to be correct, we seem to have overestimated the shear stiffness of the asfaboard. In fact if we compare the measured result to the theory using the previously

discussed results from [30] we get a much better agreement:

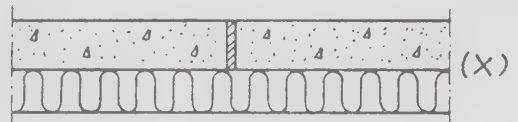
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	$2.6E+10$	$1.8E+07$	$2.6E+10$
G-modulus	G [Pa]		$5.0E+06$	
Density	ρ [kg/m ³]	2300	800	2300
Poiss. ratio	ν [1]	0.25	0.30	0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.012	



B 14

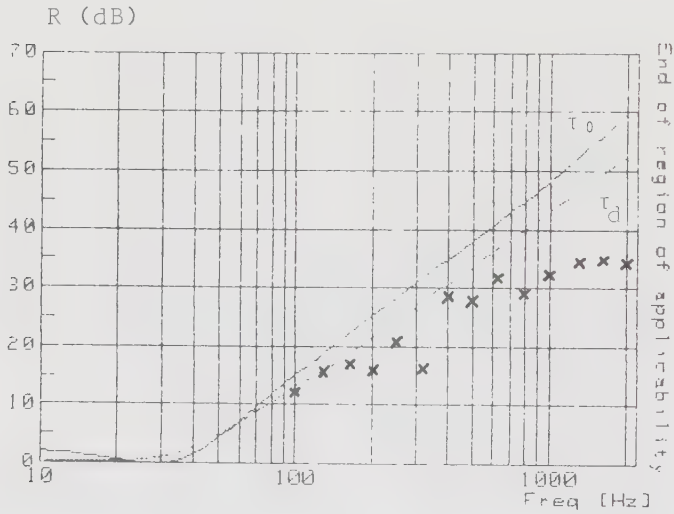
Foundation: Mineral wool (70 mm)
Interlayer: Single asfboard (13 mm)



We also consider the case of the cellular plastic as an interlayer:

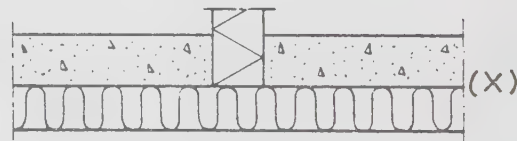
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	$2.6E+10$	$9.0E+06$	$2.6E+10$
G-modulus	G [Pa]		$2.0E+06$	
Density	Rho [kg/m ³]	2300	30	2300
Poiss. ratio	ν_y [1]	0.25		0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.080	



B 18

Foundation: Mineral wool (70 mm)
Interlayer: Cellular plastic (80 mm)



We see that the agreement is not perfect, but still acceptable up to about 600 Hz, where the measured curve changes its gradient. We may remind ourselves that the theory only considers thin elastic layers, thin in the meaning that the width does not exceed half a wavelength. The actual wavelength is the transverse or the shear wavelength

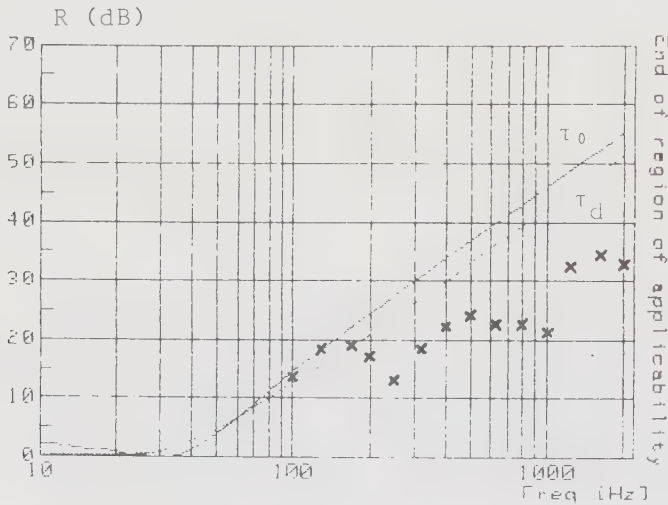
$$\lambda_S = \sqrt{G/\rho} / f \quad (1)$$

For the cellular plastic we have $l = \lambda_S/2$ at about 1600 Hz, so it may be the wave character of the inter-layer that is the cause of the high frequency deviation in the figure above. The corresponding frequency f_S for the asf aboard is about 11 kHz, for the 52 mm asf aboard strips about 800 Hz and for the 50 mm mineral wool about 900 Hz.

Below we show the case of the 50 mm mineral wool. We do not know the value of the effective static load during the measurement, and this is an important parameter for the correct choice of the modules. We have here chosen to use the values corresponding to a 2.4 kPa static load:

MATERIALS AND DIMENSIONS

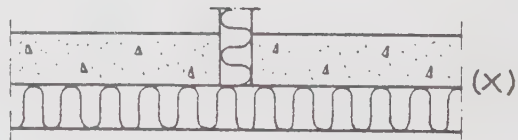
ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	5.0E+05	2.6E+10
G-modulus	G [Pa]		1.3E+06	
Density	Rho[kg/m ³]	2300	160	2300
Poiss. ratio	My [1]	0.25		0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.050	



B 36

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (50 mm)

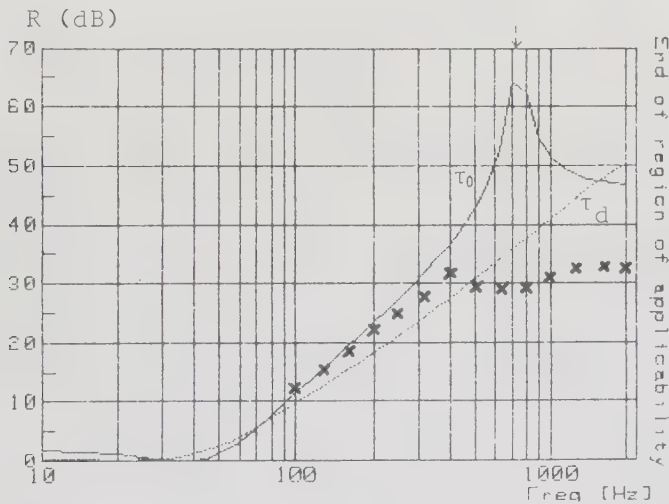


The agreement with theory is not very good. We may expect a resonance phenomenon at $f_S \approx 900$ Hz, and this does indeed appear. However we would expect better agreement for frequencies below that.

We also show the case of 52 mm stripped asfaboard:

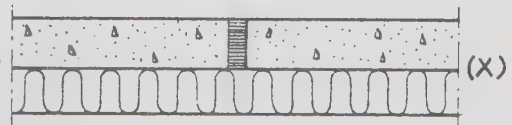
MATERIALS AND DIMENSIONS

ELEMENT No:		1	2	3
E-modulus	E [Pa]	2.6E+10	6.0E+07	2.6E+10
G-modulus	G [Pa]		2.0E+06	
Density	Rho[kg/m ³]	2300	160	2300
Poiss. ratio	My [1]	0.25		0.25
Thickness	H [m]	0.080	0.080	0.080
Width of layer	L [m]		0.052	



B 16

Foundation: Mineral wool (70 mm)
 Interlayer: Striped asfaboard (26 mm)



Here we have good agreement up to about 500 Hz where the curve changes character abruptly. This may be in good accordance with the calculated frequency $f_S \approx 800$ Hz.

We do not show any further examples, but we may point out that the larger widths of mineral wool deviate more from the theoretical curve than the 50 mm one. The wave medium behaviour of the interlayer starts here at lower frequencies. A special difficulty in the case of the mineral wool is that we do not know exactly the value of the effective static load during the measurement. As the modules of mineral wool depend strongly on this parameter, the values of those are difficult to estimate.

Finally we may point out, that the effect of constraints at the junction is not at all in accordance with our theoretical calculations. We would have expected a considerable improvement for a large single sided mass load, but this is not experienced. Instead we get a very irregular curve with a lower transmission loss at high frequencies than before. For the case of a mass load on both sides of the junction we do experience a certain improvement, but the curveform is not in accordance with our theory.

3.6 Summary and concluding remarks to Part 3

In part 3.2 we present the transmission loss measurements for different types of junctions (see Appendix I). A comparison in pairs of several different results is also made (see Appendix II). The main conclusions we can draw from this comparison study are as follows:

The type of foundation (A or B) does not affect the transmission loss much. However, a continuous foundation of type A (cellular plastic) may cause a somewhat higher transmission than an interrupted one.

The asfaboard turned out to be a poor interlayer, but a considerable improvement was achieved by using the foundation materials, A or B, as interlayers (10 - 15 dB improvement compared to the asfaboard). Similar improvement was also achieved for the asfaboard in strips stacked in the junction. This measure reduces the shear stiffness but probably increases the longitudinal stiffness compared to an equal width of the original asfaboard.

Then we consider increasing widths of mineral wool as an interlayer, and we find that the transmission loss is improved when the width increases. The improvement is however more moderate than we might have expected.

The final comparison series considers the influence of different constraints at the junction, which we also considered in the theoretical work. However the measured results are not all in agreement with the theoretical considerations.

In part 3.3 we present measurements of the properties of the test materials. We measure a higher G-modulus

than E-modulus for both the asfaboard and the mineral wool, and this is explained by the anisotropy of the materials. Another important observation is the strong dependence on the applied static load for the modules of the mineral wool.

In part 3.4 we first present the few field measurements, and then a method for resonance damping of the slabs is suggested. Measurement results are shown, but the conclusion is that the method is probably too complex and expensive compared to the relatively moderate improvements.

In part 3.5 we make several comparisons between theory and measurements. The general result seems to be that for low frequencies where the interlayer can be considered to be thin, it is possible to get an acceptable agreement. The experienced deviations may to a certain extent be explained by inaccurate material properties.

At higher frequencies the transmission loss is seen to change character. This is explained by the wave medium behaviour of the interlayer, which is not considered in our theoretical calculations.

For an increasing width of an interlayer the wave character starts at a successively lower frequency. This may explain the behaviour of the curves in the measurement series for different widths of the mineral wool.

For the case of different constraints at the junction, we can only state that the measurements disagree with our theoretical considerations. No satisfactory explanation for this discrepancy has yet been found.

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APPENDIX I

APPENDIX I Transmission loss measurements

I.1-9 80 mm cellular plastic (A) as a foundation.

INTERLAYER	FOUNDATION	"ID"
.1 Single asfboard (13 mm)	A	A2
.2 Double asfboard (2x13 mm)	A	A4
.3 Cellular plastic (80 mm)	A	A25
.4 Airgap (13 mm)	A	A3
.5 Slabs in direct contact	A	A26
.6 Single asfboard (13 mm)	A,continuous	A17
.7 Airgap (13 mm)	A,continuous	A24
.8 Single asfboard (13 mm)	A	A16
(Extra load, 150 kg/m on each side of the junction.)		
.9 Double asfboard (2x13 mm)	A	A5
(No compression during the measurement.)		

I.10-24 70 mm mineral wool (B) as a foundation.

INTERLAYER	FOUNDATION	EXTRA LOAD	"ID"
.10 Single asfboard (13 mm)	B		B14
.11 Double asfboard (2x13 mm)	B		B15
.12 Cellular plastic (80 mm)	B		B18
.13 Mineral wool (70 mm)	B		B12
.14 Stripped asfboard (26 mm)	B		B16
.15 Stripped asfboard (52 mm)	B		B17
.16 Airgap (13 mm)	B		B10
.17 Mineral wool (50 mm)	B		B36
.18 Mineral wool (2x50 mm)	B		B35
.19 Mineral wool (4x50 mm)	B		B33
.20 Mineral wool (50 mm)	B	1x150kg/m	B37
.21 Mineral wool (50 mm)	B	1x430kg/m	B38
.22 Mineral wool (50 mm)	B	2x280kg/m	B43
.23 Asfboard (13 mm)	B	1x430kg/m	B39
.24 Asfboard (13 mm)	B	2x280kg/m	B44

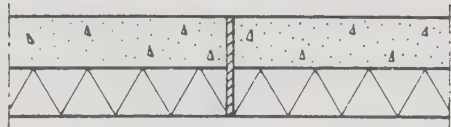
I.25-27 Field measurements

- .25 100 mm concrete slabs
 foundation: 50 mm mineral wool (150 kg/m³)
 interlayer: 50 mm cellular plastic (20 kg/m³)

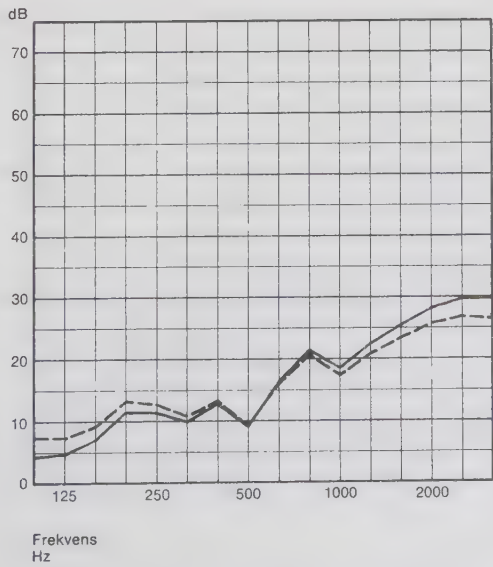
- .26 80 mm concrete slabs with reinforcing beams
foundation: 40 mm mineral wool
foundation at the junction: 40 mm cellular plastic
interlayer: airgap
- .27 Same as .26 except the foundation at the junction
is interrupted instead of continuous

A 2

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Single asfaboard (13 mm)



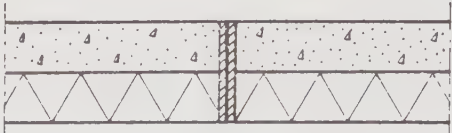
Level difference (ΔL)----
Transmission loss (R) ———



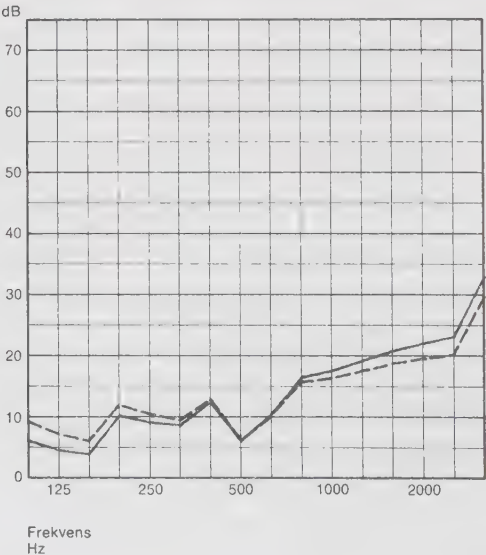
Frekvens Hz	R dB
100	4.1
125	4.5
160	6.0
200	11.3
250	11.2
315	9.7
400	12.7
500	9.0
630	16.2
800	21.4
1000	18.5
1250	22.4
1600	25.4
2000	28.2
2500	29.7
3150	29.0

A4

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Double asfboard
(2 x 13 mm)



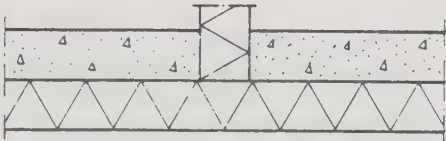
Level difference (ΔL) ----
Transmission loss (R) ———



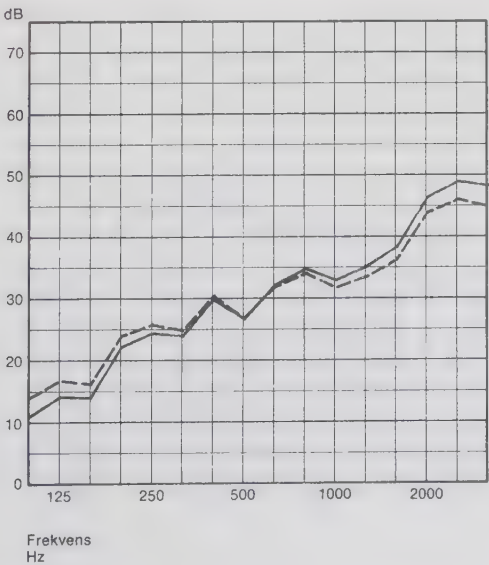
Frekvens Hz	R dB
100	5.0
125	4.2
160	3.6
200	9.9
250	8.0
315	8.3
400	12.2
500	5.9
630	10.2
800	16.3
1000	17.4
1250	19.0
1600	20.6
2000	21.9
2500	23.0
3150	32.0

A 25

Foundation: Cellular plastic (80 mm)
continuous
Interlayer: Cellular plastic (80 mm)



Level difference (ΔL) ----
Transmission loss (R) —

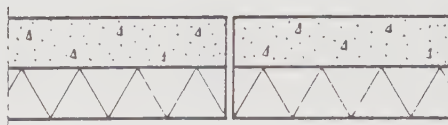


Frekvens Hz	R dB
100	10.6
125	13.0
160	13.7
200	21.9
250	24.1
315	23.6
400	29.6
500	26.4
630	31.9
800	34.6
1000	32.7
1250	34.9
1600	38.0
2000	46.1
2500	48.6
3150	47.9

A3

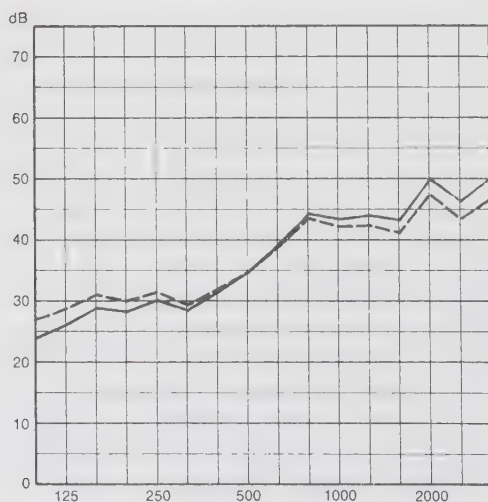
Foundation: Cellular plastic (80 mm)
interrupted

Interlayer: Airgap (13 mm)



Level difference (ΔL) ----

Transmission loss (R) —



Frekvens
Hz

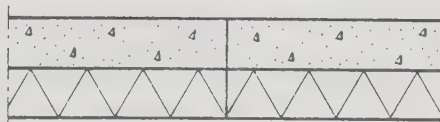
Frekvens Hz	R dB
100	23.5
125	25.7
160	28.5
200	27.9
250	29.8
315	28.1
400	31.2
500	34.4
630	38.9
800	44.0
1000	43.1
1250	43.7
1600	42.9
2000	49.7
2500	46.1
3150	49.9

A 26

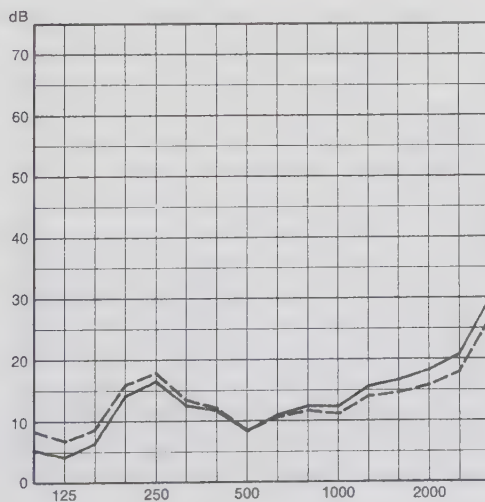
Foundation: Cellular plastic (80 mm)

Interlayer: None

Note: Slabs in direct contact

Level difference (ΔL) ----

Transmission loss (R) —

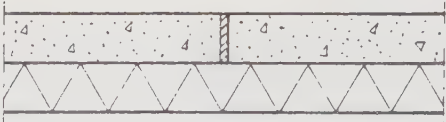
Frekvens
Hz

Frekvens Hz	R dB
100	5.0
125	3.8
160	6.1
200	13.8
250	16.2
315	12.2
400	11.4
500	8.1
630	10.7
800	12.2
1000	12.1
1250	15.4
1600	16.4
2000	18.1
2500	20.6
3150	29.6

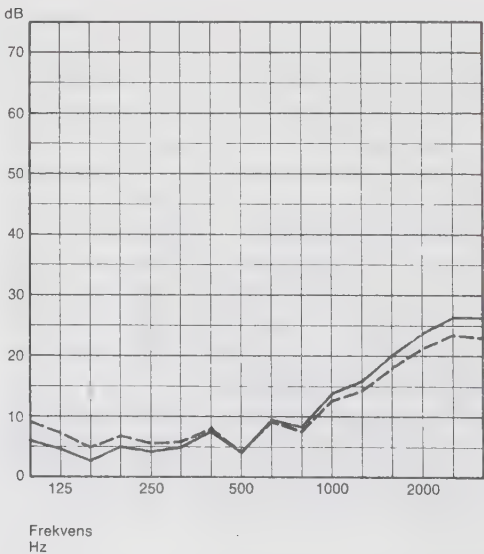
TEKNISKA HÖGSKOLAN I LUND Byggnadsakustik	I.6	A-17	KURVBLAD NR	
			MATDAG 82:12:21	
			MAT LP	RIT /HP
	GRANSKAD			

A 17

Foundation: Cellular plastic (80 mm)
 continuous
Interlayer: Single asfaboard (13 mm)



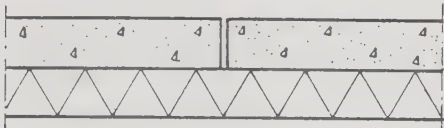
Level difference (ΔL) ----
Transmission loss (R) ———



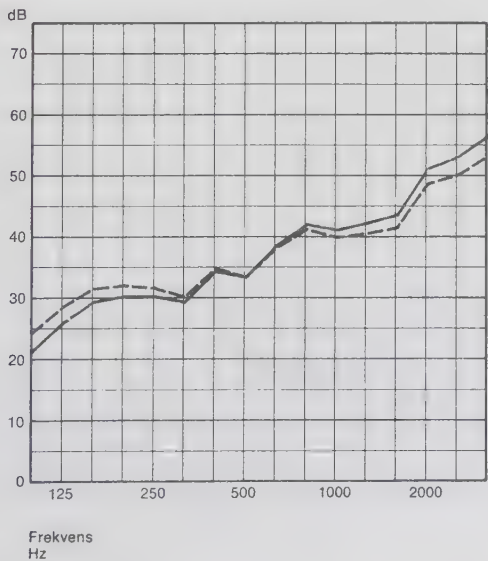
Frekvens Hz	R dB
100	5.7
125	4.3
160	2.4
200	4.7
250	3.9
315	4.6
400	7.3
500	3.8
630	9.2
800	8.1
1000	13.6
1250	15.7
1600	19.9
2000	23.6
2500	26.2
3150	26.1

A 24

Foundation: Cellular plastic (80 mm)
 continuous
Interlayer: Airgap (13 mm)



Level difference (ΔL)-----
Transmission loss (R) ———



Frekvens Hz	R dB
100	20.7
125	25.3
160	28.0
200	29.7
250	29.7
315	28.7
400	33.8
500	32.0
630	37.9
800	41.4
1000	40.5
1250	41.6
1600	42.9
2000	50.5
2500	52.4
3150	55.9

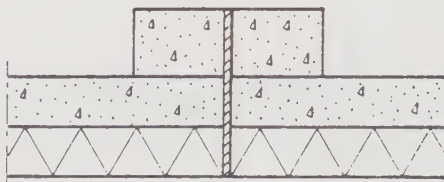
TEKNISKA HÖGSKOLAN I LUND Byggnadsakustik	1.8	A-16	KURVBLAD NR
			MÅTDAG 83:10:06
	SIGN LP		/HP

A16

Foundation: Cellular plastic (80 mm)
interrupted

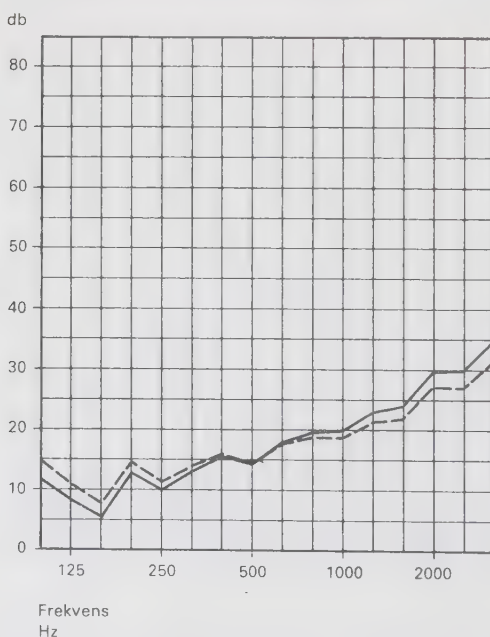
Interlayer: Single asfaboard (13 mm)

Extra load: 150 kg/m on each side
of the junction



Level difference (ΔL) ----

Transmission loss (R) —



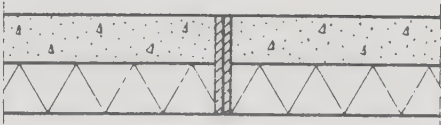
Frekvens Hz	R dB
100	11.4
125	8.0
160	5.3
200	12.5
250	9.7
315	12.7
400	15.2
500	14.0
630	17.6
800	19.2
1000	19.5
1250	22.5
1600	23.4
2000	29.1
2500	29.3
3150	34.1

A5

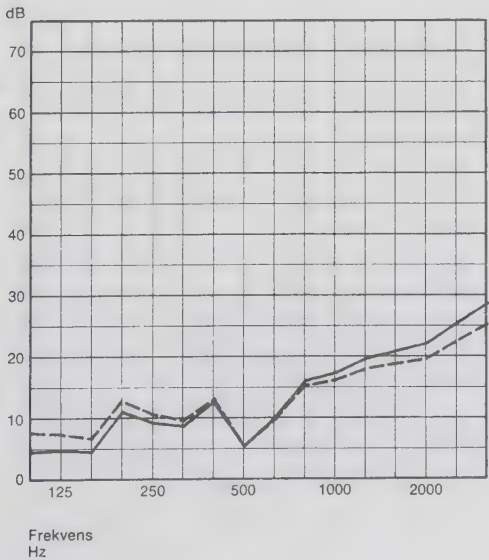
Foundation: Cellular plastic (80 mm)
interrupted

Interlayer: Double asfaboard
(2 x 13 mm)

Note: No compression during
the measurement



Transmission loss (ΔL) ----
Level difference (R) —

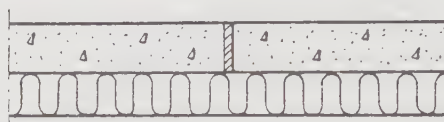


Frekvens Hz	R dB
100	4.2
125	4.4
160	4.2
200	10.7
250	8.9
315	8.3
400	12.3
500	5.1
630	9.7
800	15.0
1000	17.1
1250	19.4
1600	20.6
2000	21.9
2500	25.2
3150	26.4

B 14

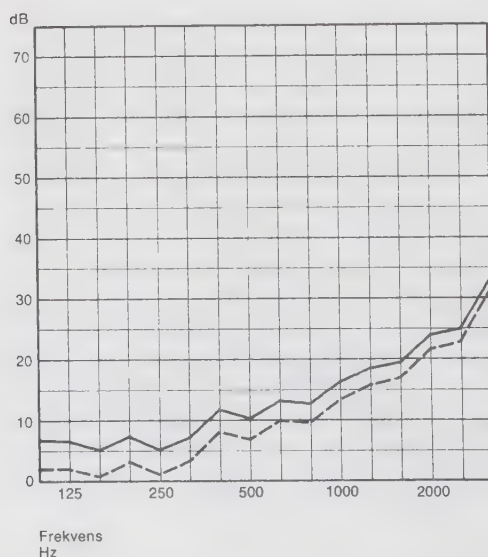
Foundation: Mineral wool (70 mm)

Interlayer: Single asfaboard (13 mm)



Level difference (ΔL)----

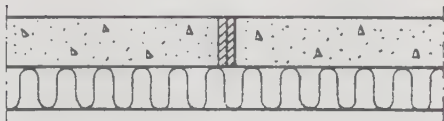
Transmission loss (R) ———



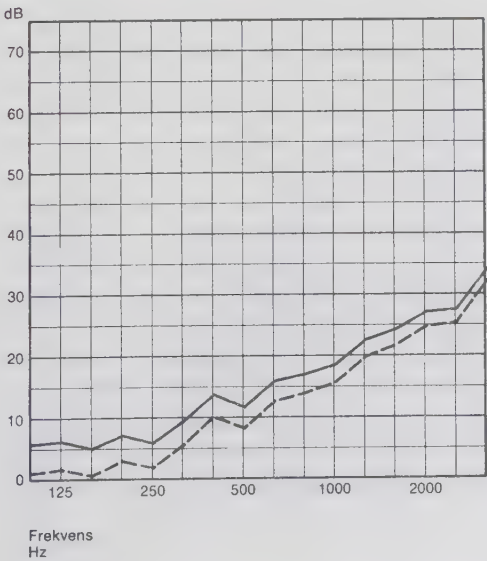
Frekvens Hz	R dB
100	6.7
125	6.5
160	5.1
200	7.3
250	5.2
315	7.2
400	11.8
500	10.3
630	13.2
800	12.8
1000	16.4
1250	18.6
1600	19.6
2000	24.1
2500	25.2
3150	33.5

B 15

Foundation: Mineral wool (70 mm)
Interlayer: Double asfaboard
(2 x 13 mm)



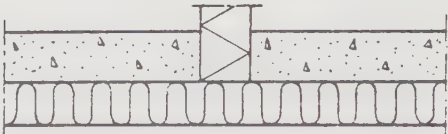
Level difference (ΔL) ----
Transmission loss (R) ———



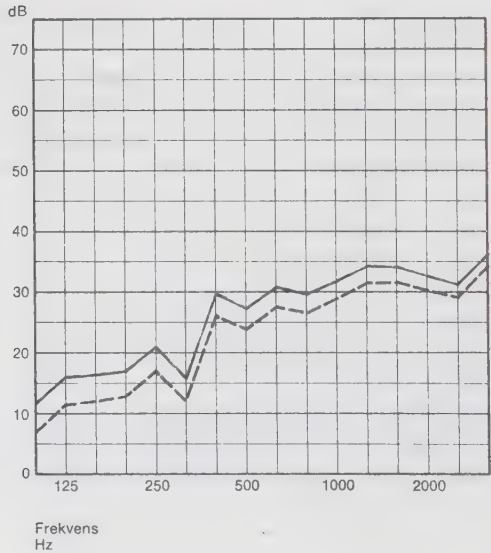
Frekvens Hz	R dB
100	5.4
125	5.8
160	4.6
200	6.7
250	5.5
315	9.0
400	13.3
500	11.2
630	15.4
800	16.6
1000	18.1
1250	22.1
1600	23.8
2000	26.6
2500	27.1
3150	33.6

B 18

Foundation: Mineral wool (70 mm)
Interlayer: Cellular plastic (80 mm)



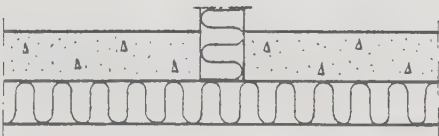
Level difference (ΔL) ----
Transmission loss (R) ———



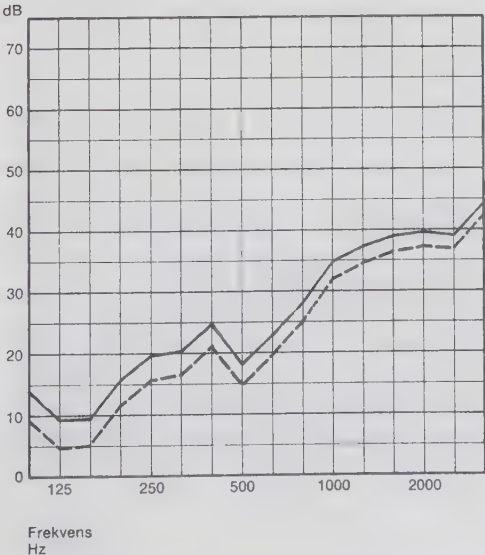
Frekvens Hz	R dB
100	11.4
125	15.0
160	16.2
200	16.0
250	20.9
315	15.7
400	29.6
500	27.2
630	30.7
800	29.6
1000	31.0
1250	34.2
1600	34.1
2000	32.6
2500	31.3
3150	36.3

B 12

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (70 mm)



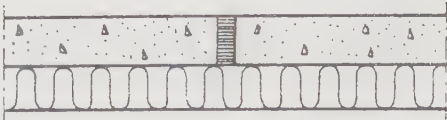
Level difference (ΔL) ----
Transmission loss (R) —



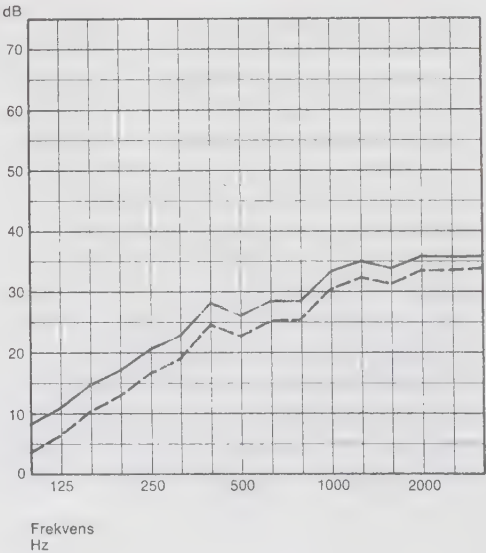
Frekvens Hz	R dB
100	13.8
125	9.1
160	9.3
200	15.6
250	19.6
315	20.3
400	24.7
500	18.1
630	22.8
800	28.1
1000	34.9
1250	37.3
1600	39.0
2000	39.6
2500	39.1
3150	44.4

B 16

Foundation: Mineral wool (70 mm)
Interlayer: Stripped asfaboard (26 mm)



Level difference (ΔL) ----
Transmission loss (R) ——

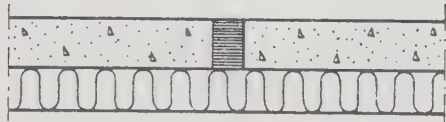


Frekvens Hz	R dB
100	8.2
125	10.9
160	14.7
200	17.1
250	20.7
315	22.9
400	28.3
500	26.2
630	28.6
800	28.7
1000	33.6
1250	35.3
1600	34.1
2000	36.1
2500	36.1
3150	36.2

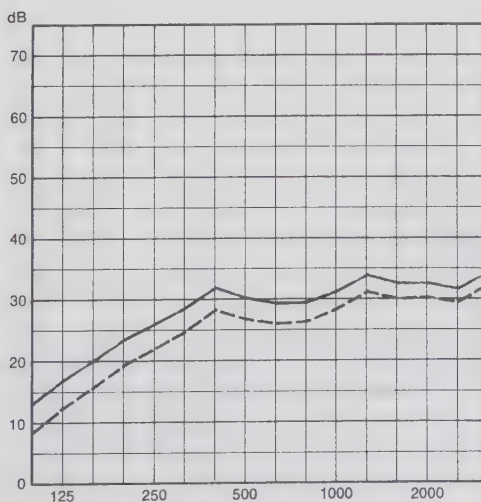
B 17

Foundation: Mineral wool (70 mm)

Interlayer: Stripped asfboard (52 mm)

Level difference (ΔL)----

Transmission loss (R) ———

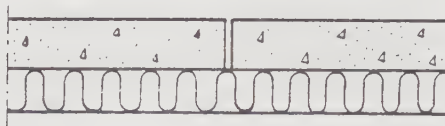
Frekvens
Hz

Frekvens Hz	R dB
100	12.7
125	16.5
160	19.7
200	23.1
250	25.7
315	28.3
400	31.6
500	29.9
630	29.0
800	29.2
1000	31.0
1250	33.6
1600	32.3
2000	32.3
2500	31.4
3150	34.1

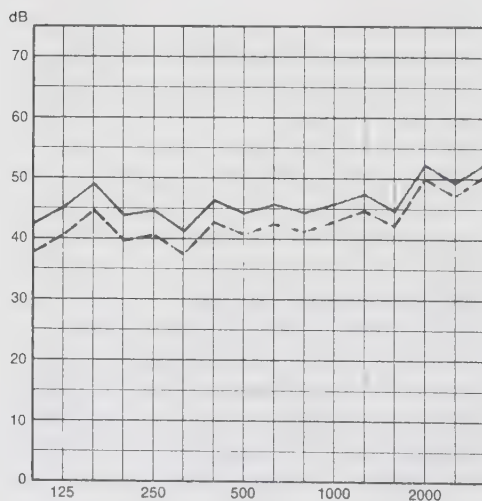
B 10

Foundation: Mineral wool (70 mm)

Interlayer: Airgap (13 mm)

Level difference (ΔL)----

Transmission loss (R) ———

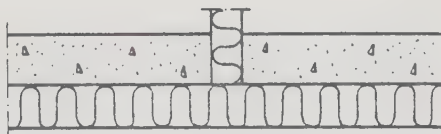
Frekvens
Hz

Frekvens Hz	R dB
100	42.3
125	45.0
160	48.9
200	43.7
250	44.6
315	41.2
400	46.3
500	44.2
630	45.7
800	44.4
1000	45.9
1250	47.5
1600	44.8
2000	52.4
2500	49.5
3150	52.7

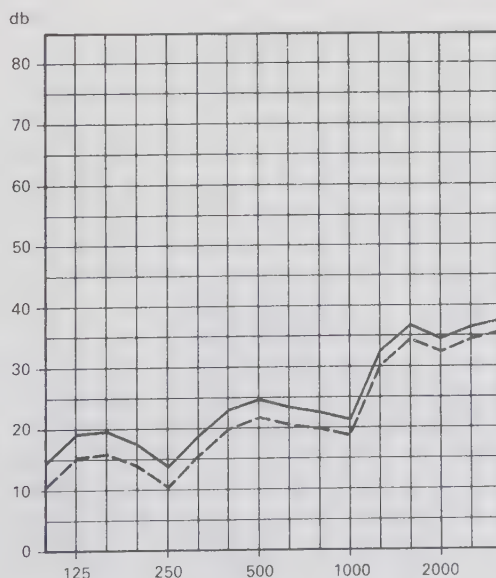
B 36

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (50 mm)

Level difference (ΔL) ----

Transmission loss (R) ———

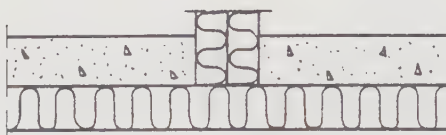
Frekvens
Hz

Frekvens Hz	R dB
100	14.3
125	18.9
160	19.2
200	17.1
250	13.4
315	18.4
400	22.6
500	24.2
630	22.9
800	22.1
1000	20.8
1250	31.9
1600	36.0
2000	33.8
2500	35.7
3150	36.7

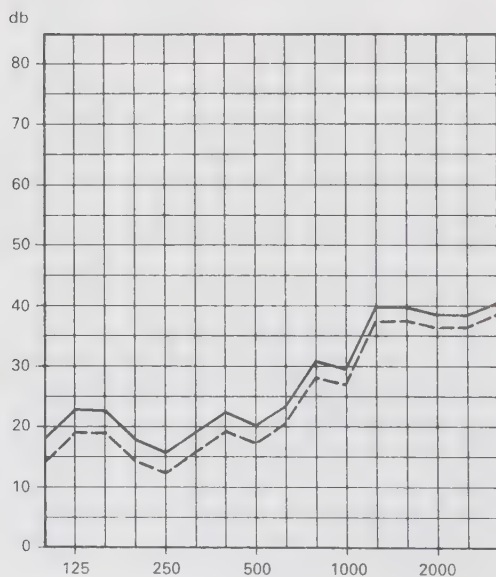
B 35

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (2 x 50 mm)

Level difference (ΔL) -----

Transmission loss (R) -----

Frekvens
Hz

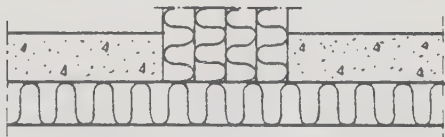
Frekvens Hz	R dB
100	18.1
125	22.0
160	22.5
200	17.0
250	15.6
315	19.0
400	22.3
500	20.1
630	23.3
800	30.7
1000	29.3
1250	39.5
1600	39.4
2000	38.2
2500	38.1
3150	40.0

TEKNISKA HÖGSKOLAN ILUND Byggnadsakustik	I.19	B-33	KURVBLAD NR
			MÄTDAG 83:06:13
			SIGN LP /HP

B 33

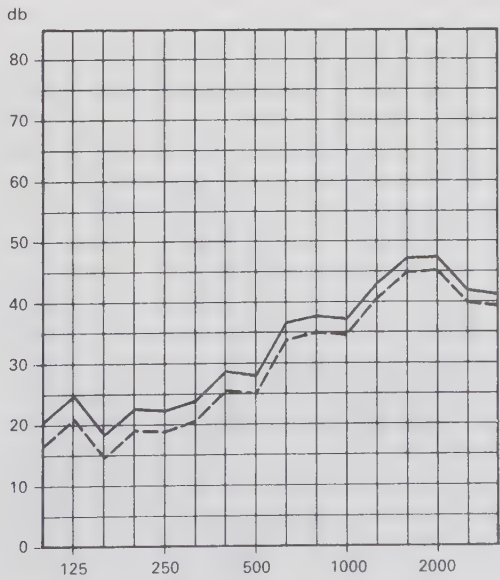
Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (4 x 50 mm)



Level difference (ΔL) ----

Transmission loss (R) ———



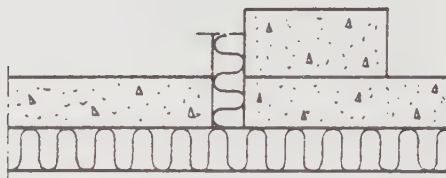
Frekvens
Hz

Frekvens Hz	R dB
100	20.3
125	24.5
160	18.1
200	22.3
250	21.9
315	23.5
400	28.3
500	27.5
630	36.1
800	37.2
1000	36.6
1250	42.3
1600	46.4
2000	46.6
2500	41.2
3150	40.4

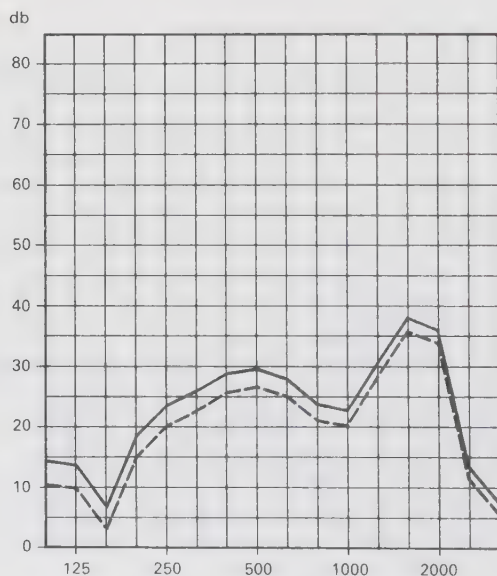
B 37

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (50 mm)

Extra load: 150 kg/m on one side
of the junctionLevel difference (ΔL) -----

Transmission loss (R) -----

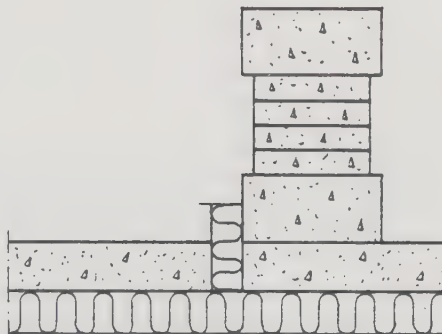
Frekvens
Hz

Frekvens Hz	R dB
100	14.2
125	13.5
160	6.6
200	18.3
250	23.2
315	25.6
400	28.5
500	29.2
630	27.6
800	23.5
1000	22.4
1250	30.2
1600	37.5
2000	35.5
2500	13.2
3150	7.2

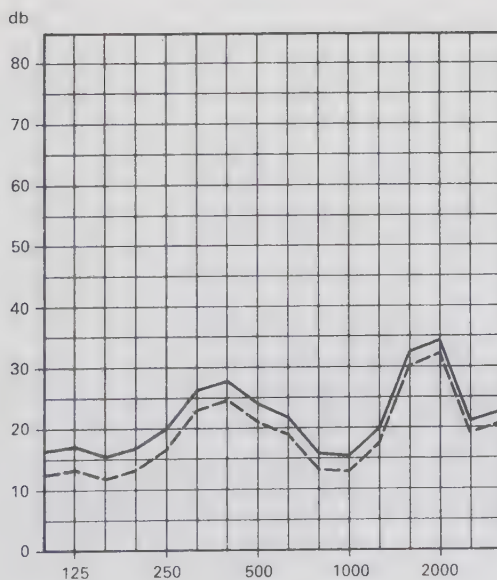
B 38

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (50 mm)

Extra load: 430 kg/m on one side
of the junctionLevel difference (ΔL) -----

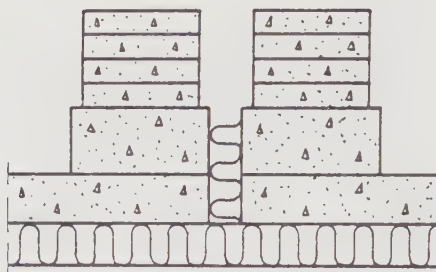
Transmission loss (R) -----

Frekvens
Hz

Frekvens Hz	R dB
100	16.2
125	16.9
160	15.2
200	16.6
250	19.7
315	25.9
400	27.4
500	23.6
630	21.4
800	15.6
1000	15.1
1250	19.6
1600	31.9
2000	33.8
2500	20.7
3150	22.1

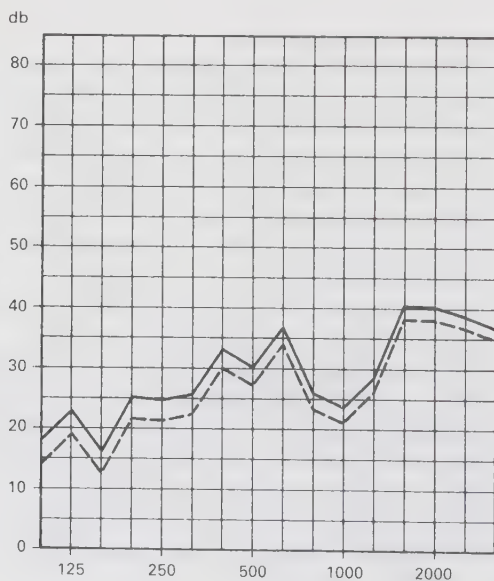
B 43

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (50 mm)
 Extra load: 280 kg/m on each side
 of the junction



Level difference (ΔL) -----

Transmission loss (R) -----

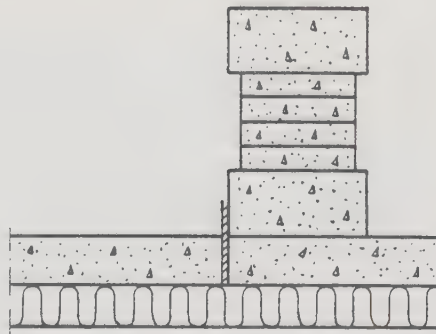


Frekvens
Hz

Frekvens Hz	R dB
100	17.9
125	22.7
160	16.0
200	24.9
250	24.4
315	25.4
400	32.8
500	29.8
630	36.4
800	25.7
1000	23.3
1250	28.2
1600	39.9
2000	39.7
2500	38.2
3150	36.2

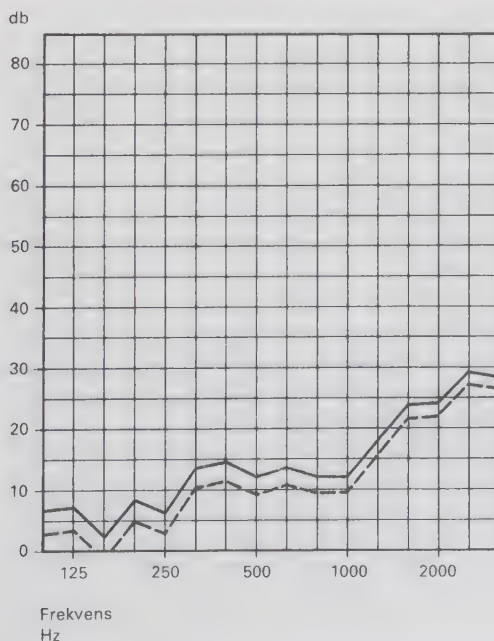
B 39

Foundation: Mineral wool (70 mm)
 Interlayer: Single asfboard (13 mm)
 Extra load: 430 kg/m on one side
 of the junction



Level difference (ΔL) - - - -

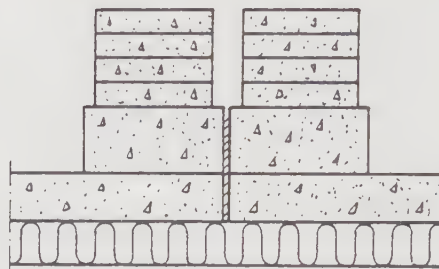
Transmission loss (R) ———



Frekvens Hz	R dB
100	6.5
125	7.0
160	2.1
200	8.1
250	5.9
315	13.2
400	14.2
500	11.7
630	13.2
800	11.7
1000	11.6
1250	17.5
1600	23.2
2000	23.5
2500	28.5
3150	27.6

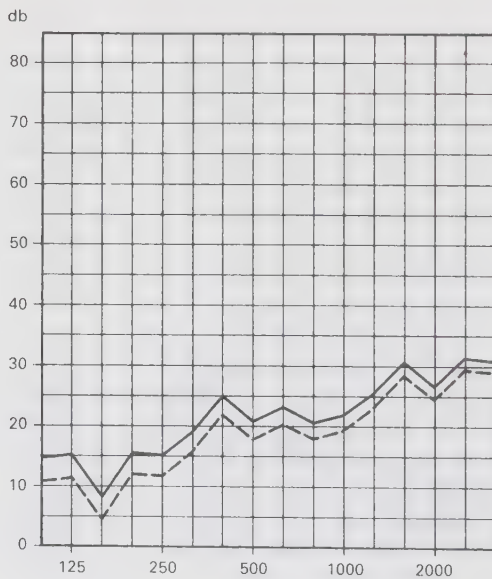
B 44

Foundation: Mineral wool (70 mm)
 Interlayer: Asfaboard (13 mm)
 Extra load: 280 kg/m on each side
 of the junction



Level difference (ΔL) ----

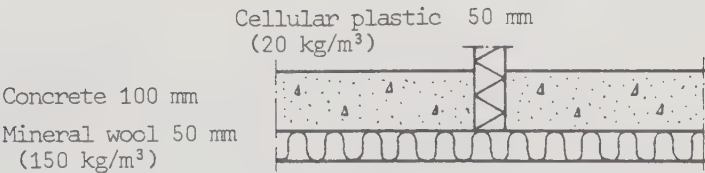
Transmission loss (R) ———



Frekvens
Hz

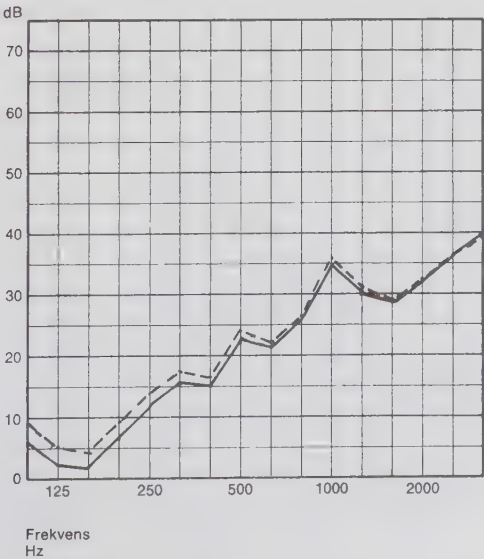
Frekvens Hz	R dB
100	14.6
125	15.1
160	8.1
200	15.4
250	14.9
315	18.0
400	24.7
500	20.5
630	22.9
800	20.4
1000	21.6
1250	25.2
1600	30.2
2000	26.2
2500	30.9
3150	30.3

F 10



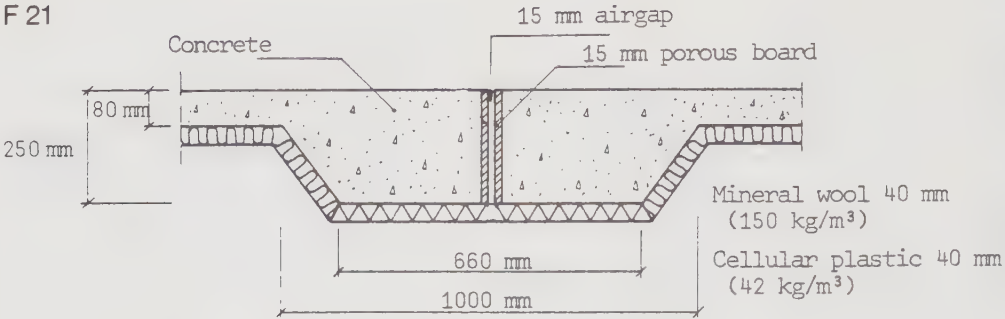
Level difference (ΔL) - - - -

Transmission loss (R) ————



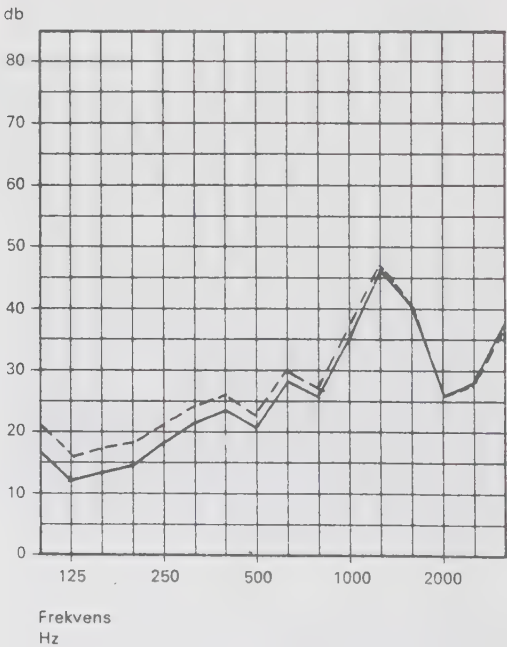
Frekvens Hz	R dB
100	5.8
125	2.4
160	2.2
200	6.9
250	11.8
315	15.5
400	15.0
500	22.9
630	21.0
800	26.2
1000	34.8
1250	30.1
1600	28.8
2000	32.2
2500	36.0
3150	39.3

F 21



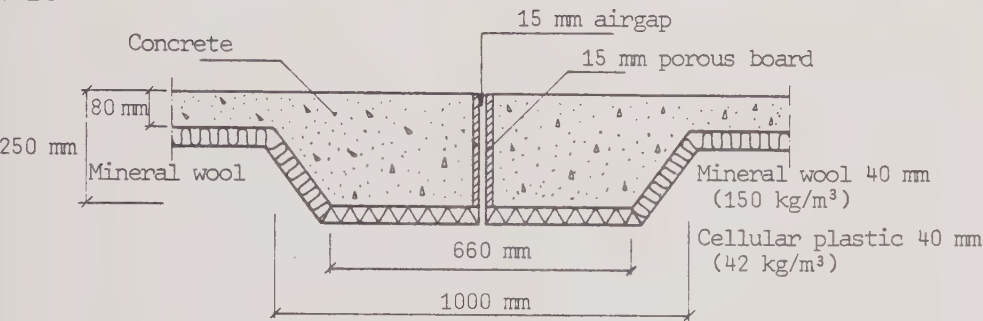
Level difference (ΔL) - - - -

Transmission loss (R) ————

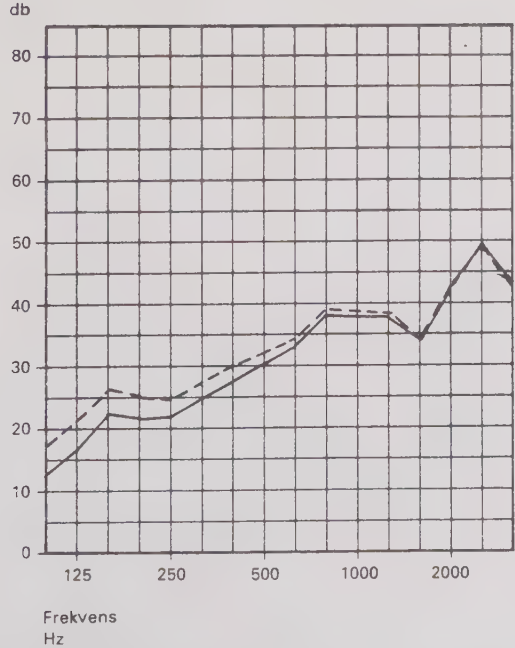


Frekvens Hz	R dB
100	16.7
125	11.8
160	13.6
200	14.8
250	18.5
315	21.4
400	23.9
500	20.5
630	28.4
800	25.7
1000	35.3
1250	46.4
1600	40.5
2000	26.0
2500	28.7
3150	37.6

F 20



Level difference (ΔL) - - - -
Transmission loss (R) ————



Frekvens Hz	R dB
100	12.7
125	16.9
160	22.6
200	21.6
250	21.8
315	24.9
400	27.7
500	30.2
630	33.1
800	38.0
1000	37.5
1250	37.5
1600	33.1
2000	42.9
2500	49.9
3150	43.3

APPENDIX II

1

APPENDIX II Comparison of different transmission loss measurements

II.1 Different foundations

	INTERLAYER	FOUNDATION	"ID"
.1	airgap	A/A continous	A3-A24
.2	asfaboard	A/A continous	A2-A17
.3	airgap	B/A	B10-A24
.4	asfaboard	A/B	A17-B14
.5	cellular plastic	A/B	A25-B18

II.2 The asfaboard as an interlayer

.1	asfaboard/direct contact	A	A2-A26
.2	airgap/asfaboard	A	A3-A2
.3	single/double asfaboard	A	A2-A4
.4	single/double asfaboard	B	B14-B15

II.3 The foundations A, B as interlayers

.1	airgap/cellular plastic	B	B10-B18
.2	airgap/mineral wool	B	B10-B12
.3	cellular plastic/mineral wool	B	B18-B12
.4	mineral wool/ asfaboard	B	B12-B14

II.4 The stripped asfaboard as an interlayer

.1	airgap/52 mm stripped asfaboard	B	B10-B17
.2	52 mm/26 mm stripped asfaboard	B	B17-B16
.3	52 mm stripped asfaboard/cellular plastic	B	B17-B18
.4	52 mm stripped asfaboard/asfaboard	B	B17-B14

II.5 Different widths of mineral wool as an inter-layer

.1	50 mm/2x50 mm mineral wool	B	B35-B36
.2	50 mm/4x50 mm mineral wool	B	B33-B36
.3	2x50 mm/4x50 mm mineral wool	B	B33-B35

II.6 Extra mass loading at the junction

	INTERLAYER	FOUNDATION	EXTRA LOAD	"ID"
.1	asfaboard	A	0/2x150kg/m	A2-A16
.2	asfaboard	B	0/2x280kg/m	B44-B14
.3	asfaboard	B	0/1x430kg/m	B39-B14
.4	mineral wool (50 mm)	B	0/1x150kg/m	B37-B36
.5	mineral wool (50 mm)	B	0/1x430kg/m	B38-B36
.6	mineral wool (50 mm)	B	0/2x280kg/m	B43-B36

II.7 Special considerations

- .1 Comparison between the case of compression and no compression during the measurement.

II.8 Field measurements

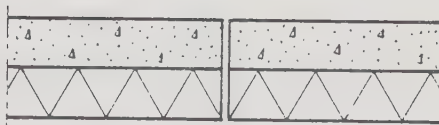
- .1 Cellular plastic interlayer in field/lab
- .2 Continuous/interrupted cellular plastic foundation in field

II.9 The effect of resonance damping

- .1 Difference in level of the receiving slab with/without the resonance damping

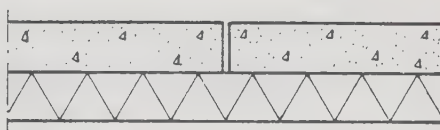
A 3

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Airgap (13 mm)

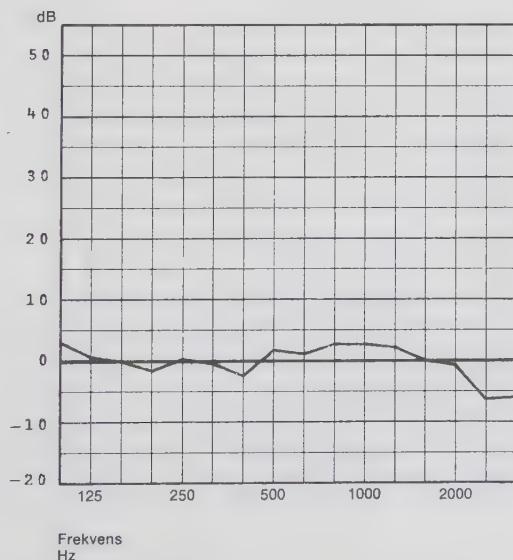


A 24

Foundation: Cellular plastic (80 mm)
continuous
Interlayer: Airgap (13 mm)



Transmission loss
difference (ΔR)



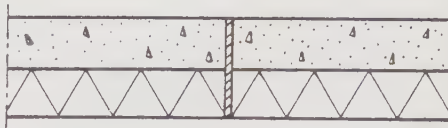
$$\Delta R = R_{A3} - R_{A24}$$

Frekvens Hz	ΔR dB
100	2.0
125	0.4
160	-0.3
200	-1.0
250	0.1
315	-0.6
400	-2.6
500	1.6
630	1.0
800	2.6
1000	2.6
1250	2.1
1600	0.0
2000	-0.0
2500	-6.3
3150	-6.0

A 2

Foundation: Cellular plastic (80 mm)
interrupted

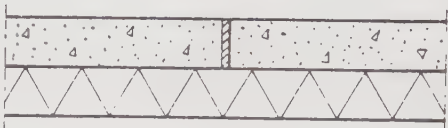
Interlayer: Single asfaboard (13 mm)



A 17

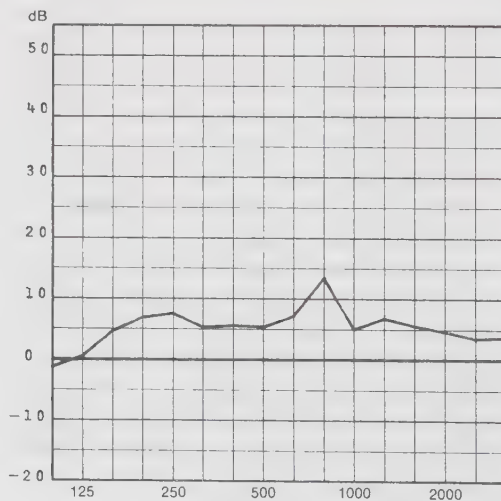
Foundation: Cellular plastic (80 mm)
continuous

Interlayer: Single asfaboard (13 mm)



Transmission loss
difference (ΔR)

$$\Delta R = R_{A:2} - R_{A:17}$$



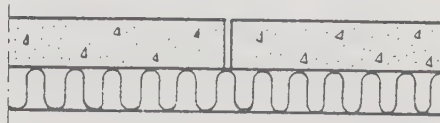
Frekvens
Hz

Frekvens Hz	ΔR dB
100	-1.6
125	0.2
160	4.4
200	6.6
250	7.3
315	5.1
400	5.4
500	5.2
630	7.0
800	13.3
1000	4.9
1250	6.7
1600	5.5
2000	4.6
2500	3.5
3150	3.7

B 10

Foundation: Mineral wool (70 mm)

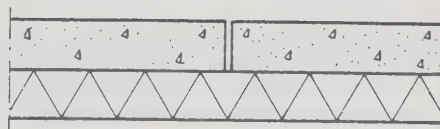
Interlayer: Airgap (13 mm)



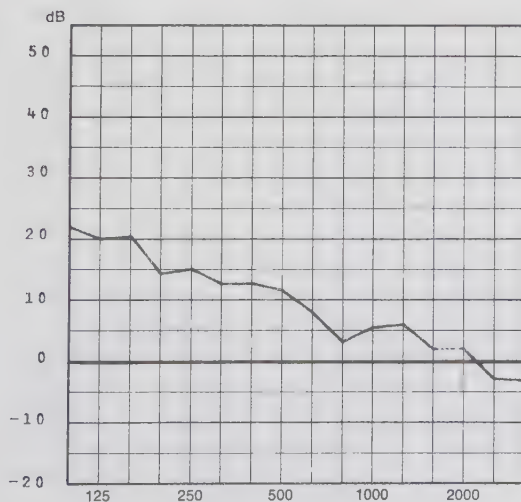
A 24

Foundation: Cellular plastic (80 mm)
continuous

Interlayer: Airgap (13 mm)

Transmission loss
difference (ΔR)

$$\Delta R = B:10 - A:24$$

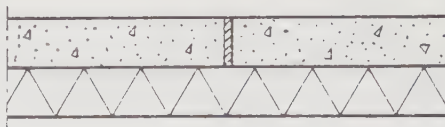
Frekvens
Hz

Frekvens Hz	ΔR dB
100	21.6
125	19.7
160	20.1
200	14.0
250	14.9
315	12.4
400	12.5
500	11.4
630	7.0
800	3.0
1000	5.4
1250	5.9
1600	1.9
2000	2.0
2500	-2.9
3150	-3.2

A 17

Foundation: Cellular plastic (80 mm)
continuous

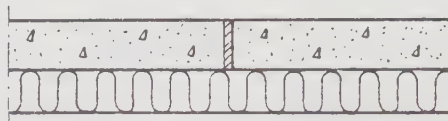
Interlayer: Single asfaboard (13 mm)



B 14

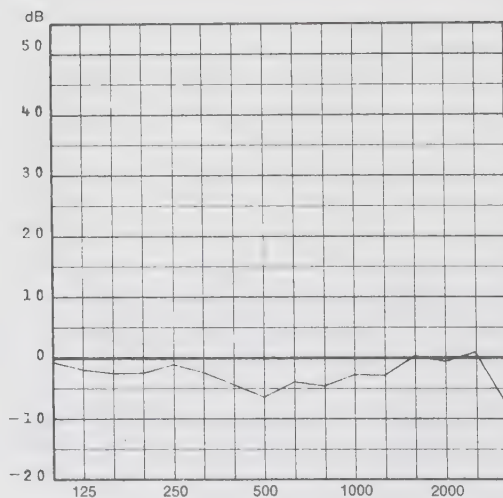
Foundation: Mineral wool (70 mm)

Interlayer: Single asfaboard (13 mm)



Transmission loss
difference (ΔR)

$$\Delta R = A:17 - B:14$$



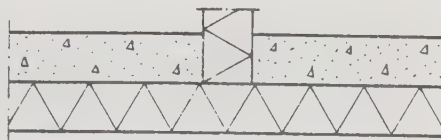
Frekvens
Hz

Frekvens Hz	ΔR dB
100	-0.0
125	-2.2
160	-2.7
200	-2.6
250	-1.3
315	-2.5
400	-4.5
500	-6.5
630	-4.0
800	-4.7
1000	-2.8
1250	-2.9
1600	0.3
2000	-0.6
2500	1.0
3150	-7.4

A 25

Foundation: Cellular plastic (80 mm)
continuous

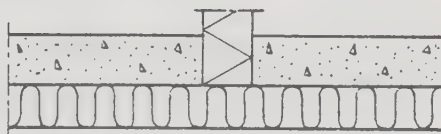
Interlayer: Cellular plastic (80 mm)



B 18

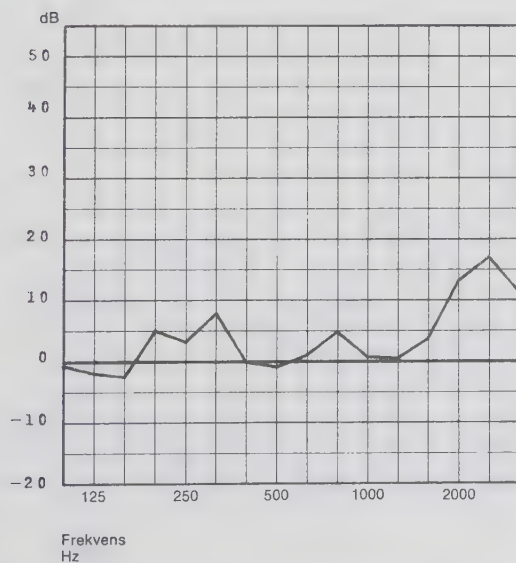
Foundation: Mineral wool (70 mm)

Interlayer: Cellular plastic (80 mm)



Transmission loss
difference (ΔR)

$\Delta R = A:25 - B:18$

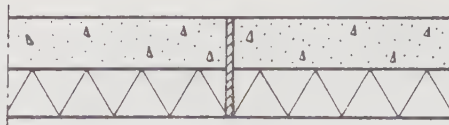


Frekvens Hz	ΔR dB
100	-2.8
125	-2.8
160	-2.5
200	5.1
250	3.2
315	8.8
400	-0.8
500	-0.8
630	1.2
800	5.8
1000	8.8
1250	0.7
1600	3.8
2000	13.4
2500	17.3
3150	11.6

A 2

Foundation: Cellular plastic (80 mm)
interrupted

Interlayer: Single asfboard (13 mm)

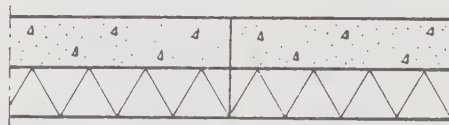


A 26

Foundation: Cellular plastic (80 mm)

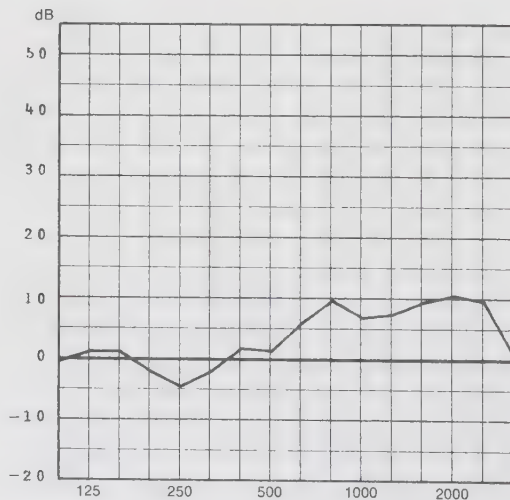
Interlayer: None

Note: Slabs in direct contact



Transmission loss
difference (ΔR)

$\Delta R = R_{A:2} - R_{A:26}$

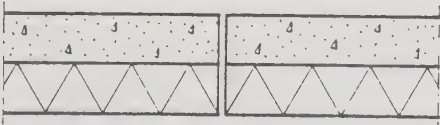


Frekvens
Hz

Frekvens Hz	ΔR dB
100	-0.9
125	0.7
160	0.7
200	-2.5
250	-5.0
315	-2.5
400	1.3
500	0.9
630	5.5
800	9.2
1000	6.4
1250	7.0
1600	9.0
2000	10.1
2500	9.1
3150	0.2

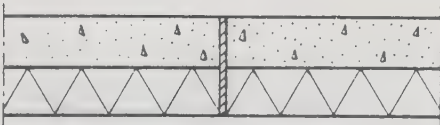
A 3

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Airgap (13 mm)



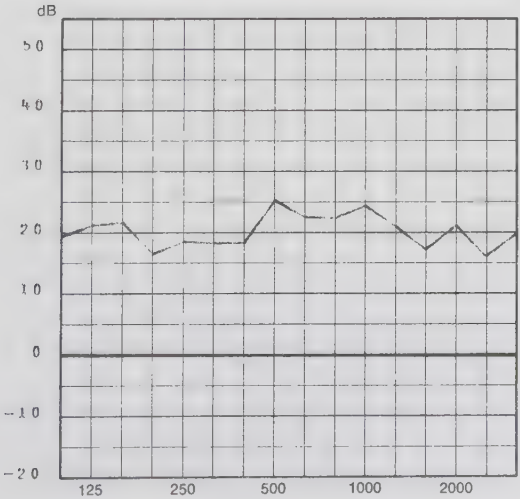
A 2

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Single asfaboard (13 mm)



Transmission loss
difference (ΔR)

$\Delta R = A_{3} - A_{2}$



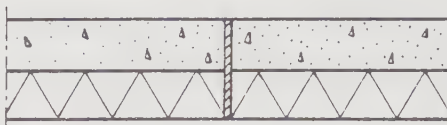
Frekvens Hz	ΔR dB
100	19.4
125	21.2
160	21.7
200	16.6
250	18.6
315	18.4
400	18.3
500	25.4
630	22.7
800	22.6
1000	24.6
1250	21.3
1600	17.5
2000	21.5
2500	16.4
3150	20.1

Frekvens
Hz

A 2

Foundation: Cellular plastic (80 mm)
interrupted

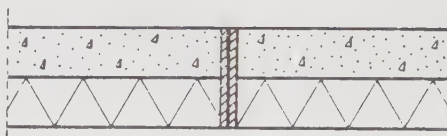
Interlayer: Single asfboard (13 mm)



A 4

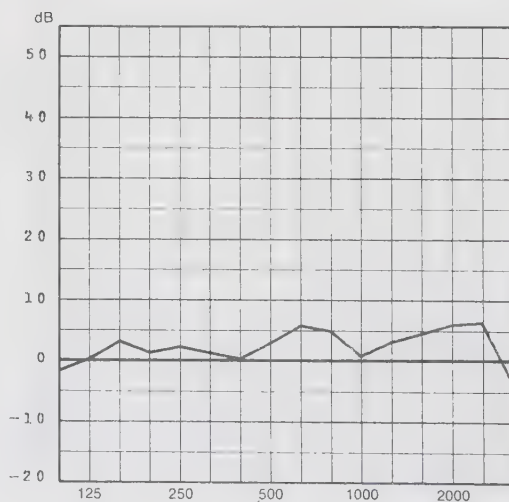
Foundation: Cellular plastic (80 mm)
interrupted

Interlayer: Double asfboard
(2 x 13 mm)



Transmission loss
difference (ΔR)

$$\Delta R = R_{A2} - R_{A4}$$

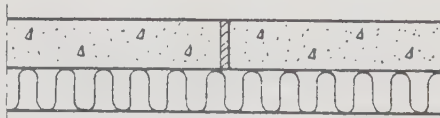


Frekvens
Hz

Frekvens Hz	ΔR dB
100	-1.7
125	0.3
160	3.2
200	1.4
250	2.4
315	1.4
400	0.5
500	3.1
630	6.0
800	5.1
1000	1.1
1250	3.4
1600	4.0
2000	6.3
2500	6.7
3150	-3.0

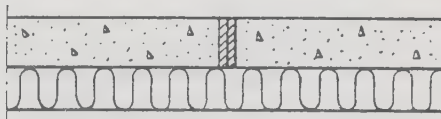
B 14

Foundation: Mineral wool (70 mm)
Interlayer: Single asfaboard (13 mm)



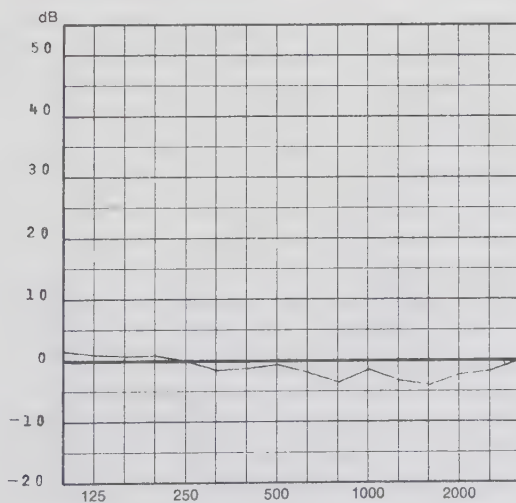
B 15

Foundation: Mineral wool (70 mm)
Interlayer: Double asfaboard
(2 x 13 mm)



Transmission loss
difference (ΔR)

$$\Delta R = B:14 - B:15$$



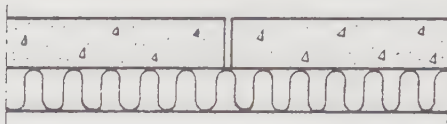
Frekvens
Hz

Frekvens Hz	ΔR dB
100	1.3
125	0.7
160	0.5
200	0.6
250	-0.3
315	-1.8
400	-1.5
500	-0.9
630	-2.2
800	-3.8
1000	-1.7
1250	-3.5
1600	-4.2
2000	-2.5
2500	-1.9
3150	-0.1

B 10

Foundation: Mineral wool (70 mm)

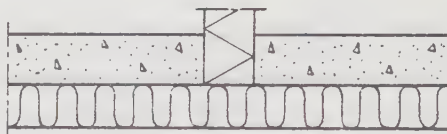
Interlayer: Airgap (13 mm)



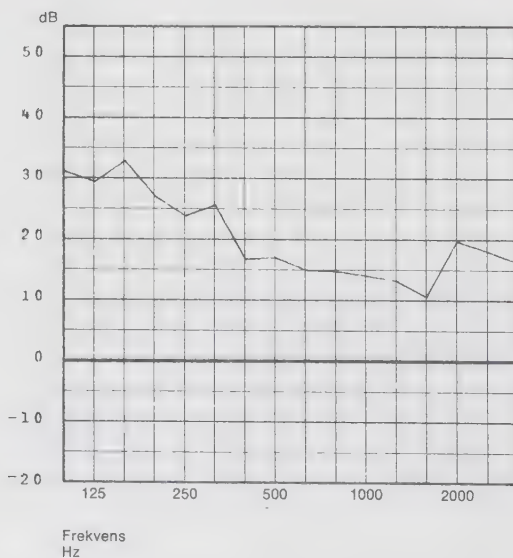
B 18

Foundation: Mineral wool (70 mm)

Interlayer: Cellular plastic (80 mm)

Transmission loss
difference (ΔR)

$$\Delta R = B:10 - B:18$$

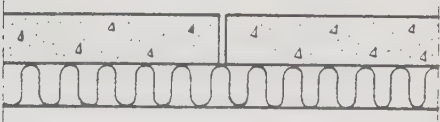


Frekvens Hz	ΔR dB
100	30.9
125	29.2
160	32.7
200	26.9
250	23.7
315	25.5
400	16.7
500	17.0
630	15.0
800	14.8
1000	14.1
1250	13.3
1600	10.7
2000	19.8
2500	18.2
3150	16.4

TEKNISKA HÖGSKOLAN I LUND Byggnadsakustik	II.3.2		KURVBLAD NR	
			MATDAG	
			MÅT	RIT
			GRANSKAD	

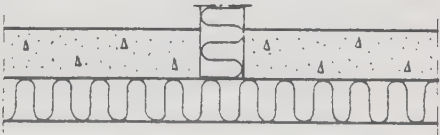
B 10

Foundation: Mineral wool (70 mm)
Interlayer: Airgap (13 mm)

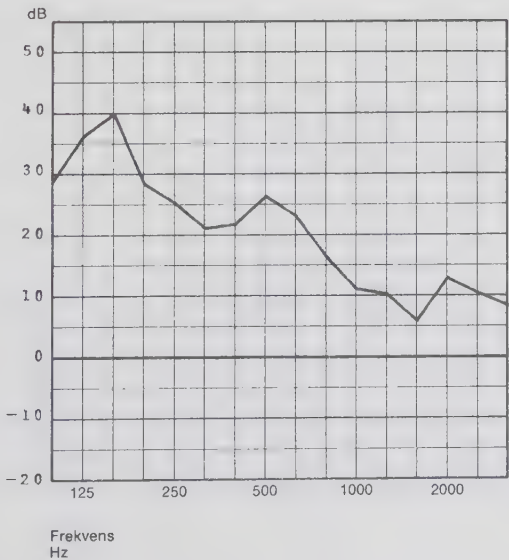


B 12

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (70 mm)



Transmission loss
difference (ΔR)



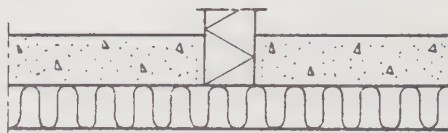
$\Delta R = B:10 - B:12$

Frekvens Hz	ΔR dB
100	28.5
125	35.9
160	39.6
200	28.1
250	25.0
315	20.9
400	21.6
500	26.1
630	22.9
800	16.3
1000	11.0
1250	10.2
1600	5.8
2000	12.8
2500	10.4
3150	8.3

B 18

Foundation: Mineral wool (70 mm)

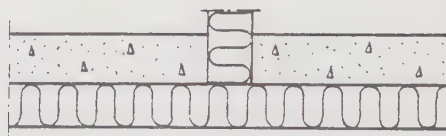
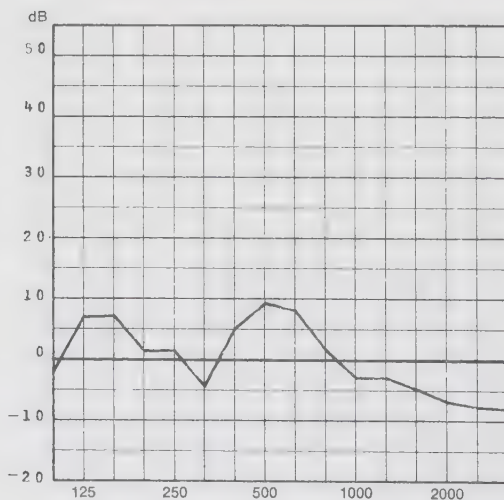
Interlayer: Cellular plastic (80 mm)



B 12

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (70 mm)

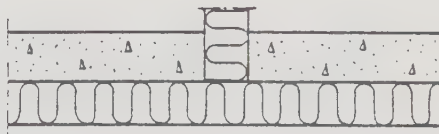
Transmission loss
difference (ΔR)Frekvens
Hz

$$\Delta R = B:18 - B:12$$

Frekvens Hz	ΔR dB
100	-2.4
125	6.7
160	6.9
200	1.2
250	1.3
315	-4.6
400	4.9
500	9.1
630	7.9
800	1.5
1000	-3.1
1250	-3.1
1600	-4.9
2000	-7.8
2500	-7.8
3150	-8.1

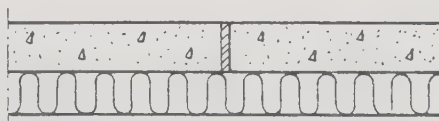
B 12

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (70 mm)



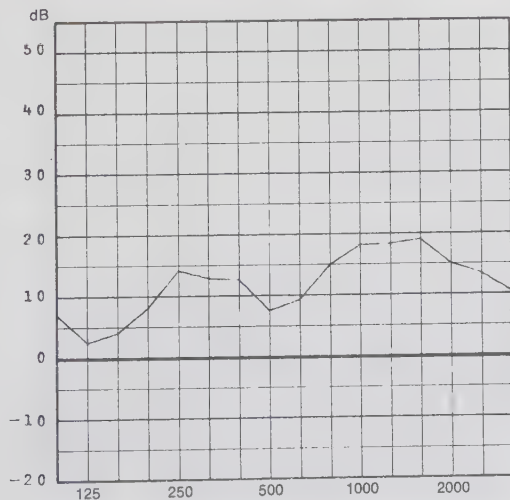
B 14

Foundation: Mineral wool (70 mm)
Interlayer: Single asfboard (13 mm)



Transmission loss
difference (ΔR)

$$\Delta R = B:12 - B:14$$



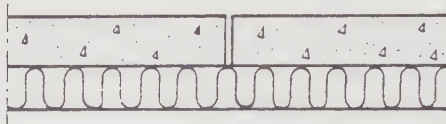
Frekvens
Hz

Frekvens Hz	ΔR dB
100	7.1
125	2.6
160	4.2
200	8.3
250	14.4
315	13.1
400	12.9
500	7.8
630	9.6
800	15.3
1000	18.5
1250	18.7
1600	19.4
2000	15.5
2500	13.9
3150	10.9

TEKNISKA HÖGSKOLAN I LUND Byggnadsakustik	11.4.1		KURVBLAD NR	
			MÄTDAG	
			MÄT	RIT
			GRANSKAD	

B 10

Foundation: Mineral wool (70 mm)
Interlayer: Airgap (13 mm)



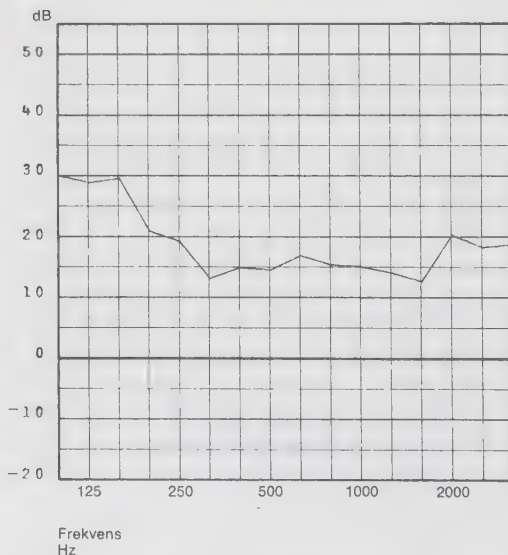
B 17

Foundation: Mineral wool (70 mm)
Interlayer: Stripped asfaboard (52 mm)



Transmission loss
difference (ΔR)

$$\Delta R = B:10 - B:17$$



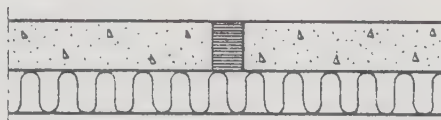
Frekvens Hz	ΔR dB
100	29.6
125	28.5
160	29.2
200	20.6
250	18.9
315	12.9
400	14.7
500	14.3
630	16.7
800	15.2
1000	14.9
1250	13.9
1600	12.5
2000	20.1
2500	18.1
3150	18.6

TEKNISKA HÖGSKOLAN I LUND Byggnadsakustik	II.4.2		KURVBLAD NR	
			MÄTDAG	
			MÅT	RIT
			GRANSKAD	

B 17

Foundation: Mineral wool (70 mm)

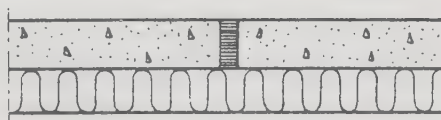
Interlayer: Stripped asfaboard (52 mm)



B 16

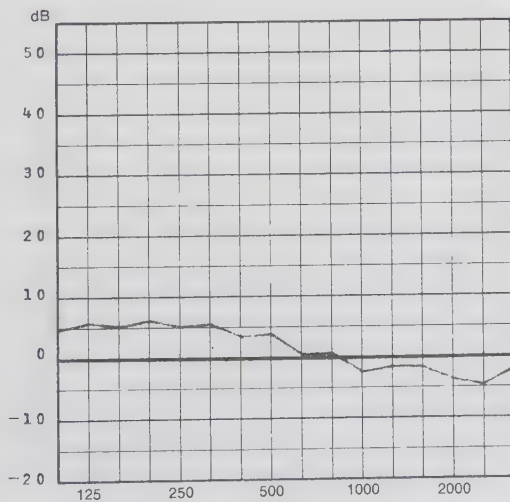
Foundation: Mineral wool (70 mm)

Interlayer: Stripped asfaboard (26 mm)



Transmission loss
difference (ΔR)

$$\Delta R = B:17 - B:16$$



Frekvens
Hz

Frekvens Hz	ΔR dB
100	4.5
125	5.6
160	5.0
200	6.0
250	5.0
315	5.4
400	3.3
500	3.7
630	0.4
800	0.5
1000	-2.6
1250	-1.7
1600	-1.0
2000	-3.8
2500	-4.7
3150	-2.1

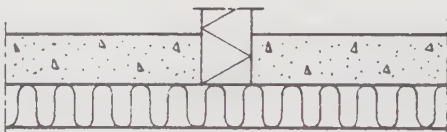
B 17

Foundation: Mineral wool (70 mm)
Interlayer: Stripped asfaboard (52 mm)



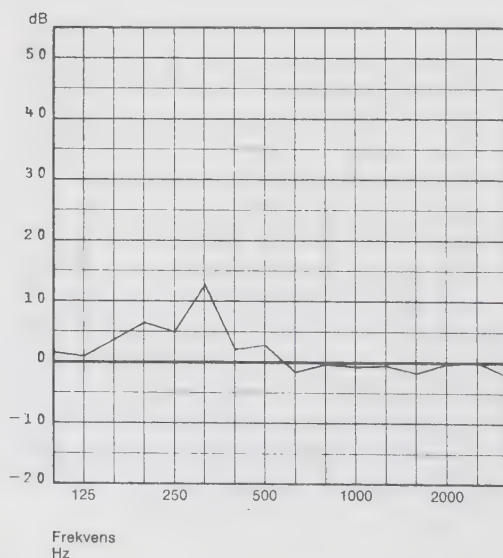
B 18

Foundation: Mineral wool (70 mm)
Interlayer: Cellular plastic (80 mm)



Transmission loss
difference (ΔR)

$$\Delta R = B:17 - B:18$$

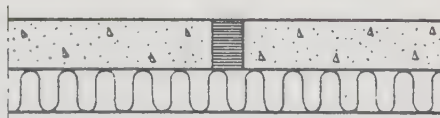


Frekvens Hz	ΔR dB
100	1.3
125	0.7
160	3.5
200	6.3
250	4.8
315	12.6
400	2.0
500	2.7
630	-1.7
800	-0.4
1000	-0.8
1250	-0.6
1600	-1.8
2000	-0.3
2500	0.1
3150	-2.2

B 17

Foundation: Mineral wool (70 mm)

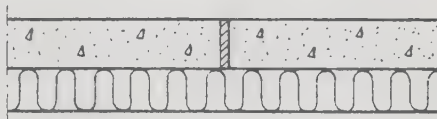
Interlayer: Stripped asfäboard (52 mm)



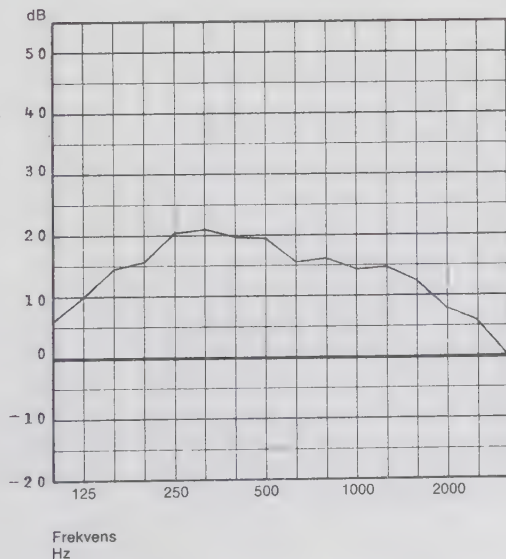
B 14

Foundation: Mineral wool (70 mm)

Interlayer: Single asfäboard (13 mm)

Transmission loss
difference (ΔR)

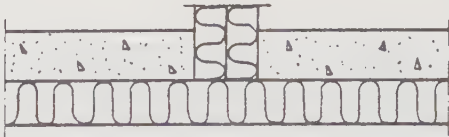
$$\Delta R = B:17 - B:14$$



Frekvens Hz	ΔR dB
100	6.0
125	10.0
160	14.6
200	15.8
250	20.5
315	21.1
400	19.8
500	19.6
630	15.8
800	16.4
1000	14.6
1250	15.0
1600	12.7
2000	8.2
2500	6.2
3150	0.6

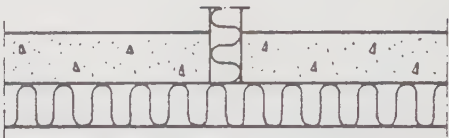
B 35

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (2 x 50 mm)



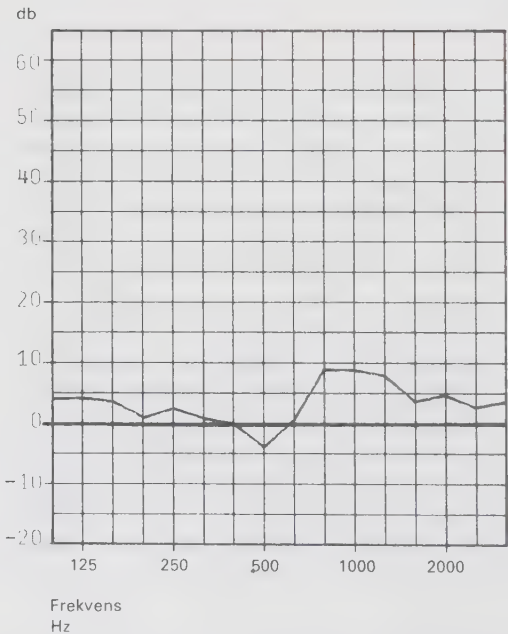
B 36

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (50 mm)



Transmission loss
difference (ΔR)

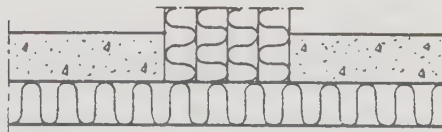
$\Delta R = B:35 - B:36$



Frekvens Hz	ΔR dB
100	3.8
125	3.9
160	3.3
200	0.7
250	2.2
315	0.6
400	-0.3
500	-4.1
630	0.4
800	8.6
1000	8.5
1250	7.6
1600	3.4
2000	4.4
2500	2.4
3150	3.3

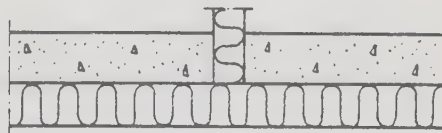
B 33

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (4 x 50 mm)



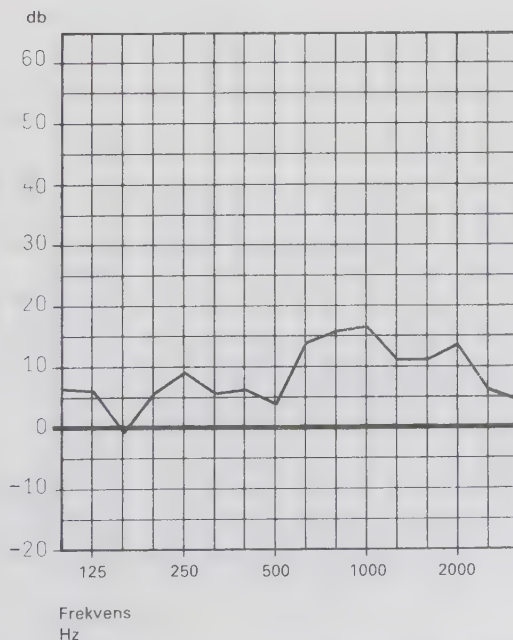
B 36

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (50 mm)



Transmission loss
 difference (ΔR)

$$\Delta R = B:33 - B:36$$



Frekvens Hz	ΔR dB
100	6.0
125	5.6
160	-1.1
200	5.2
250	8.5
315	5.1
400	5.7
500	3.3
630	13.2
800	15.1
1000	15.0
1250	10.4
1600	10.4
2000	12.0
2500	5.5
3150	3.7

TEKNISKA HÖGSKOLAN ILUND Byggnadsakustik	II.5.3	KURVBLAD NR
		MÄTDAG
		SIGN

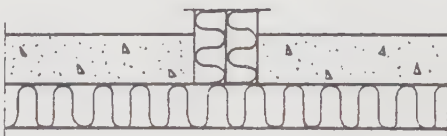
B 33

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (4 x 50 mm)



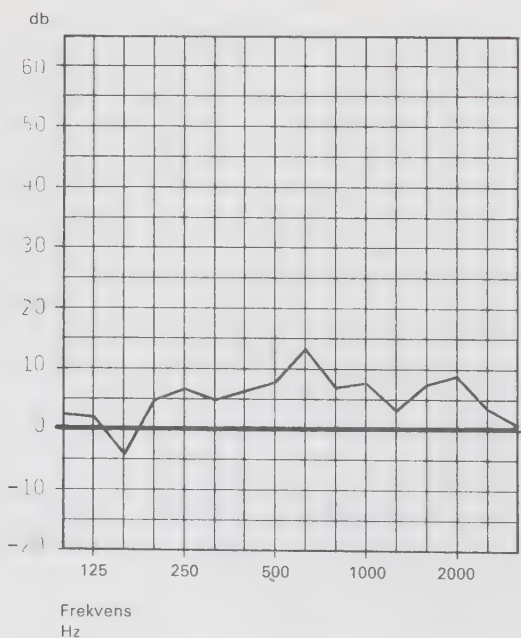
B 35

Foundation: Mineral wool (70 mm)
Interlayer: Mineral wool (2 x 50 mm)



Transmission loss
difference (ΔR)

$$\Delta R = B:33 - B:35$$

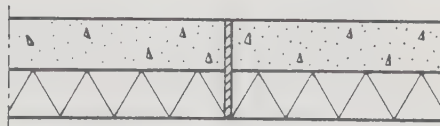


Frekvens Hz	ΔR dB
100	2.2
125	1.7
160	-4.4
200	4.5
250	6.3
315	4.5
400	6.0
500	7.4
630	12.8
800	6.5
1000	7.3
1250	2.8
1600	7.0
2000	8.4
2500	3.1
3150	0.4

A 2

Foundation: Cellular plastic (80 mm)
interrupted

Interlayer: Single asfboard (13 mm)

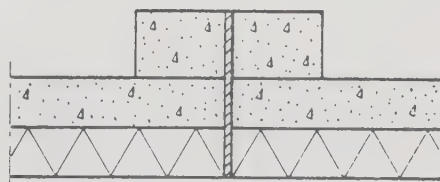


A 16

Foundation: Cellular plastic (80 mm)
interrupted

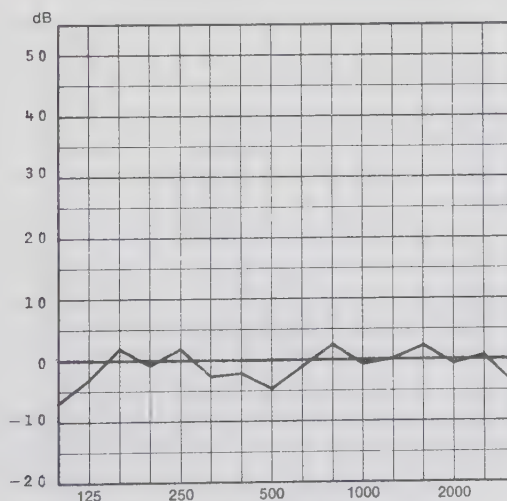
Interlayer: Single asfboard (13 mm)

Extra load: 150 kg/m on each side
of the junction



Transmission loss
difference (ΔR)

$$\Delta R = R_{A2} - R_{A16}$$



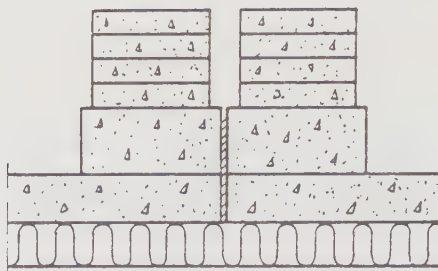
Frekvens
Hz

Frekvens Hz	ΔR dB
100	-7.3
125	-3.5
160	1.5
200	-1.2
250	1.5
315	-3.8
400	-2.5
500	-5.8
630	-1.4
800	2.2
1000	-1.8
1250	-0.1
1600	2.8
2000	-0.9
2500	0.4
3150	-4.3

TEKNISKA HÖGSKOLAN ILUND Byggnadsakustik	II.6.2		KURVBLAD NR
			MÄTDAG
			SIGN

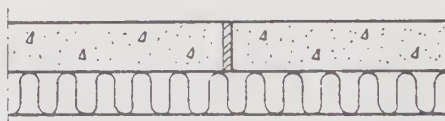
B 44

Foundation: Mineral wool (70 mm)
 Interlayer: Asfaboard (13 mm)
 Extra load: 280 kg/m on each side
 of the junction



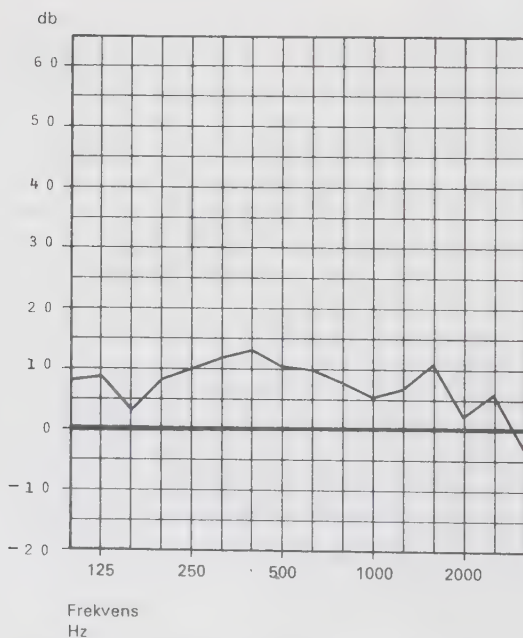
B 14

Foundation: Mineral wool (70 mm)
 Interlayer: Single asfaboard (13 mm)



Transmission loss
 difference (ΔR)

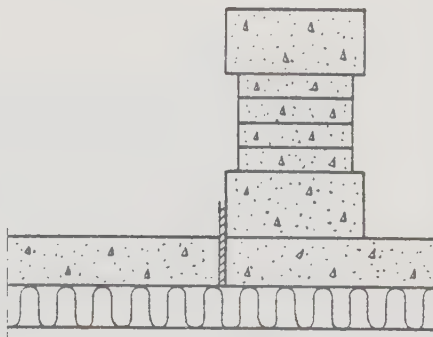
$$\Delta R = B:44 - B:14$$



Frekvens Hz	ΔR dB
100	7.9
125	8.5
160	3.8
200	8.0
250	9.8
315	11.6
400	12.9
500	10.2
630	9.6
800	7.6
1000	5.2
1250	6.6
1600	10.6
2000	2.1
2500	5.7
3150	-3.1

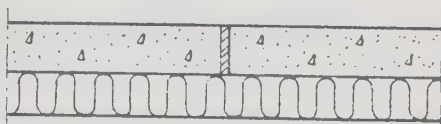
B 39

Foundation: Mineral wool (70 mm)
 Interlayer: Single asfboard (13 mm)
 Extra load: 430 kg/m on one side
 of the junction



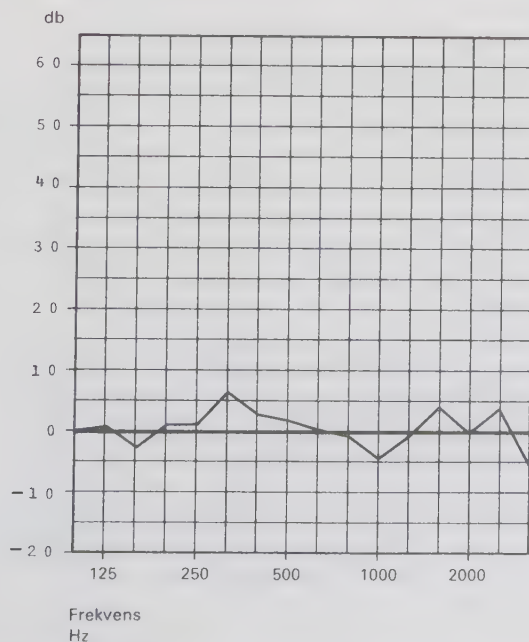
B 14

Foundation: Mineral wool (70 mm)
 Interlayer: Single asfboard (13 mm)



Transmission loss
 difference (ΔR)

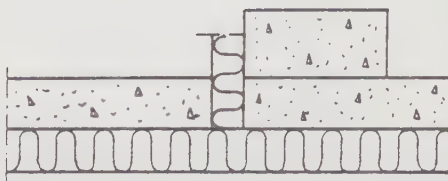
$$\Delta R = B:39 - B:14$$



Frekvens Hz	ΔR dB
100	-0.2
125	0.4
160	-3.0
200	0.7
250	0.8
315	6.0
400	2.4
500	1.4
630	-0.1
800	-1.1
1000	-4.8
1250	-1.1
1600	3.6
2000	-0.6
2500	3.3
3150	-5.8

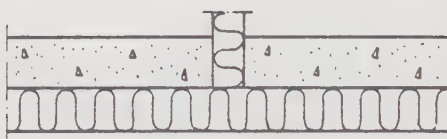
B 37

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (50 mm)
 Extra load: 150 kg/m on one side
 of the junction



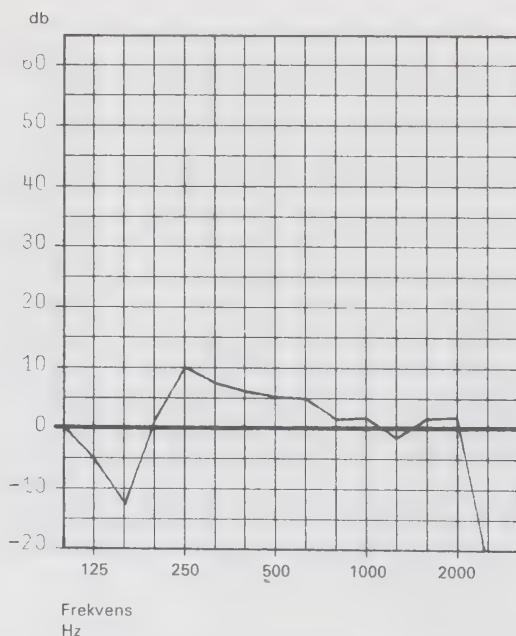
B 36

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (50 mm)



Transmission loss
 difference (ΔR)

$$\Delta R = B:37 - B:36$$

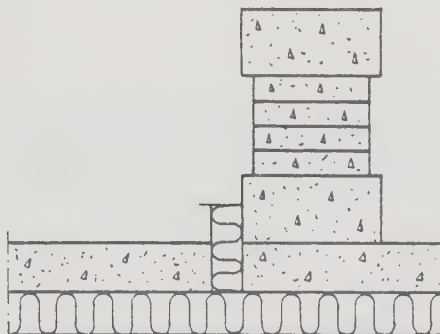


Frekvens Hz	ΔR dB
100	-0.1
125	-5.4
160	-12.6
200	1.2
250	9.8
315	7.2
400	5.9
500	5.0
630	4.7
800	1.4
1000	1.6
1250	-1.7
1600	1.5
2000	1.7
2500	-22.5
3150	-29.5

B 38

Foundation: Mineral wool (70 mm)

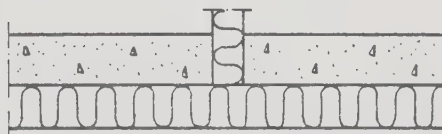
Interlayer: Mineral wool (50 mm)

Extra load: 430 kg/m on one side
of the junction

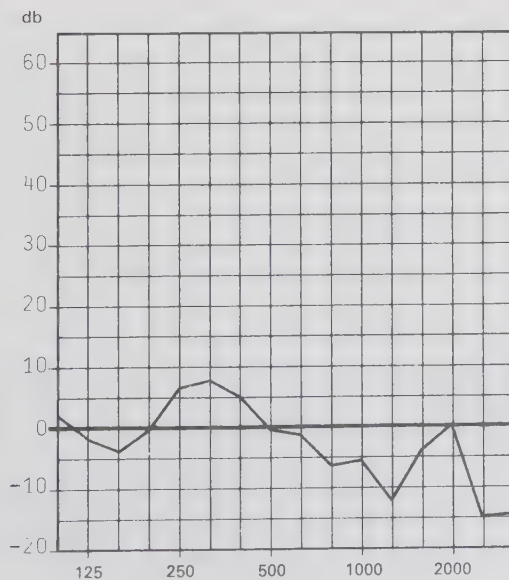
B 36

Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (50 mm)

Transmission loss
difference (ΔR)

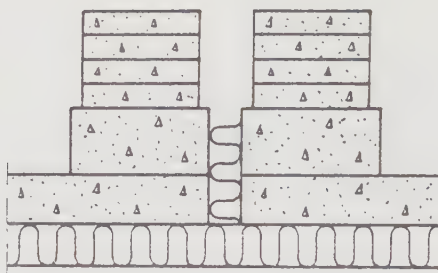
$$\Delta R = B:38 - B:36$$

Frekvens
Hz

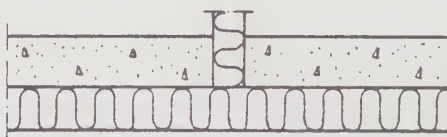
Frekvens Hz	ΔR dB
100	1.9
125	-2.0
160	-4.0
200	-0.5
250	6.3
315	7.5
400	4.8
500	-0.6
630	-1.5
800	-6.5
1000	-5.7
1250	-12.3
1600	-4.1
2000	0.0
2500	-15.0
3150	-14.6

B 43

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (50 mm)
 Extra load: 280 kg/m on each side
 of the junction

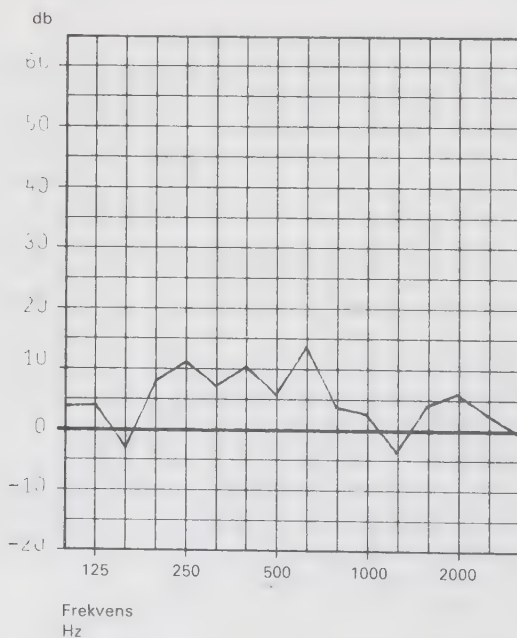
**B 36**

Foundation: Mineral wool (70 mm)
 Interlayer: Mineral wool (50 mm)



Transmission loss
 difference (ΔR)

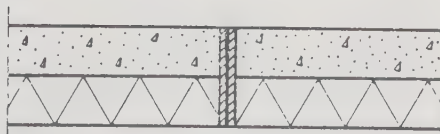
$$\Delta R = B:43 - B:36$$



Frekvens Hz	ΔR dB
100	3.6
125	3.8
160	-3.2
200	7.8
250	11.0
315	7.0
400	10.2
500	5.6
630	13.5
800	3.6
1000	2.5
1250	-3.7
1600	3.9
2000	5.9
2500	2.5
3150	-0.5

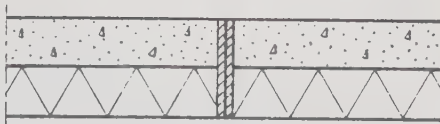
A4

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Double asfaboard
(2 x 13 mm)

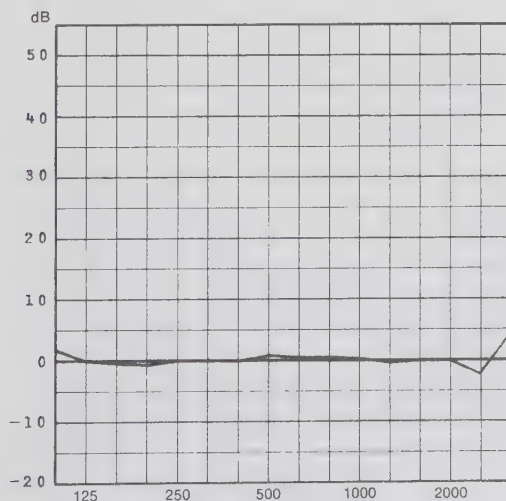


A5

Foundation: Cellular plastic (80 mm)
interrupted
Interlayer: Double asfaboard
(2 x 13 mm)
Note: No compression during
the measurement



Transmission loss
difference (ΔR)



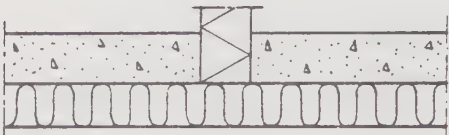
Frekvens
Hz

$$\Delta R = R_{A4} - R_{A5}$$

Frekvens Hz	ΔR dB
100	1.6
125	-0.2
160	-0.6
200	-0.8
250	-0.1
315	0.0
400	-0.1
500	0.8
630	0.5
800	0.5
1000	0.3
1250	-0.4
1600	0.0
2000	0.0
2500	-2.2
3150	4.4

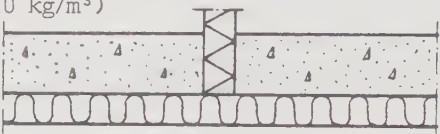
B 18

Foundation: Mineral wool (70 mm)
Interlayer: Cellular plastic (80 mm)

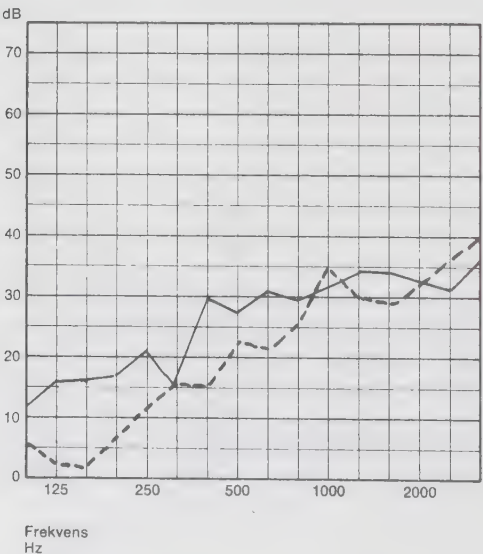


F 10

Concrete 100 mm
Mineral wool 50 mm
(150 kg/m³)
Cellular plastic 50 mm
(20 kg/m³)

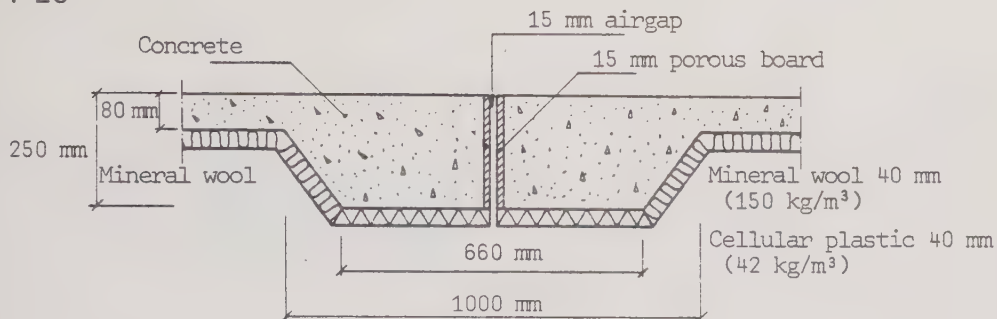


Transmission loss (R)

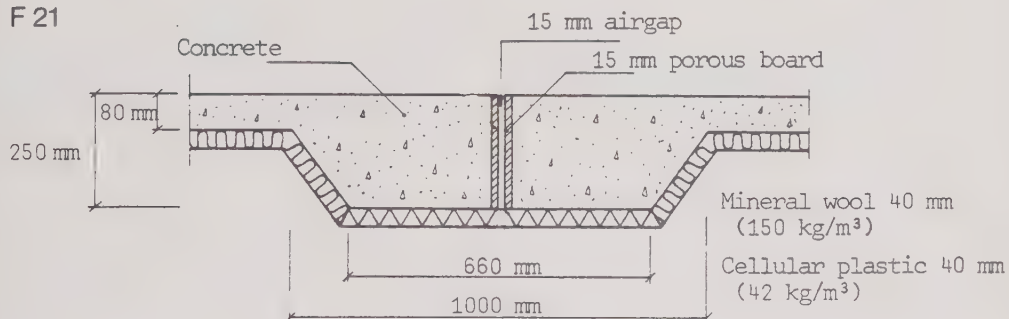


Frekvens	
Hz	dB
100	
125	
160	
200	
250	
315	
400	
500	
630	
800	
1000	
1250	
1600	
2000	
2500	
3150	

F 20

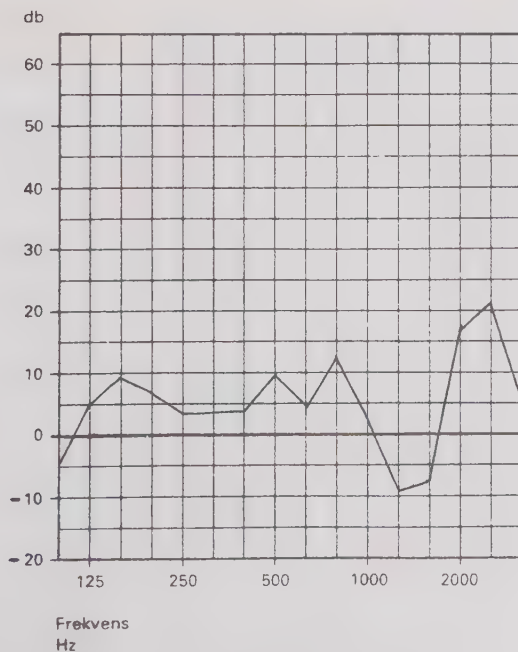


F 21



Transmission loss
difference (ΔR)

$$\Delta R = F:20 - F:21$$



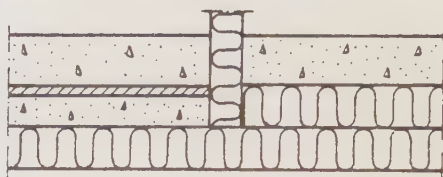
Frekvens Hz	ΔR dB
100	-4.0
125	5.1
160	9.0
200	6.8
250	3.3
315	3.5
400	3.8
500	9.7
630	4.7
800	12.3
1000	2.2
1250	-8.9
1600	-7.4
2000	16.9
2500	21.2
3150	5.7

D 41

Foundation: Mineral wool (70 mm,
2 x 70 mm respectively)

System of
damping : Concrete tiles (50 mm),
Asfaboard (14 mm)

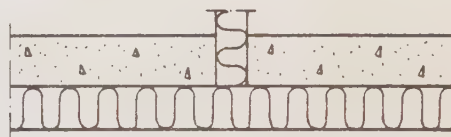
Interlayer: Mineral wool (50 mm)



B 36

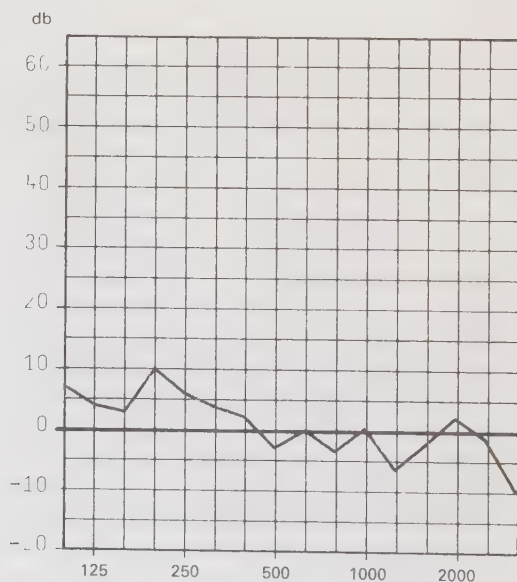
Foundation: Mineral wool (70 mm)

Interlayer: Mineral wool (50 mm)



Level difference (ΔL)

$\Delta L = D:41 - B:36$



Frekvens
Hz

Frekvens Hz	ΔL dB
100	6.9
125	3.8
160	2.7
200	9.8
250	5.0
315	3.6
400	2.0
500	-3.0
630	-0.1
800	-3.5
1000	0.3
1250	-6.4
1600	-2.3
2000	2.1
2500	-1.4
3150	-9.0

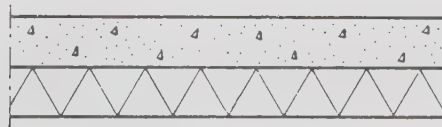
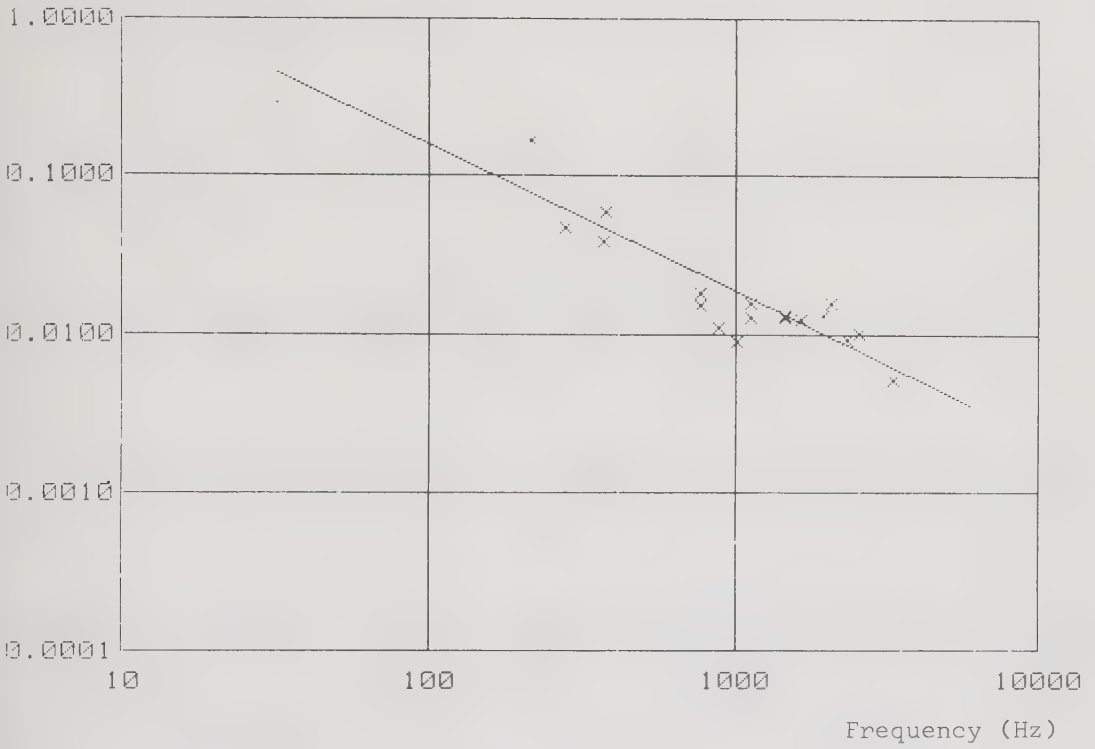
APPENDIX III

APPENDIX III Presentation of loss factor
measurements

- III.1 Laboratory slabs, foundation A
 (80 mm expanded plastic)
- .2 Laboratory slabs, foundation B
 (70 mm mineral wool)
- .3 Field measurements, slab directly on the
 ground (expanded clay)
- .4 The influence of resonance damping
- .5 Average 1/3-octave values for III.4

III.1

Loss factor η



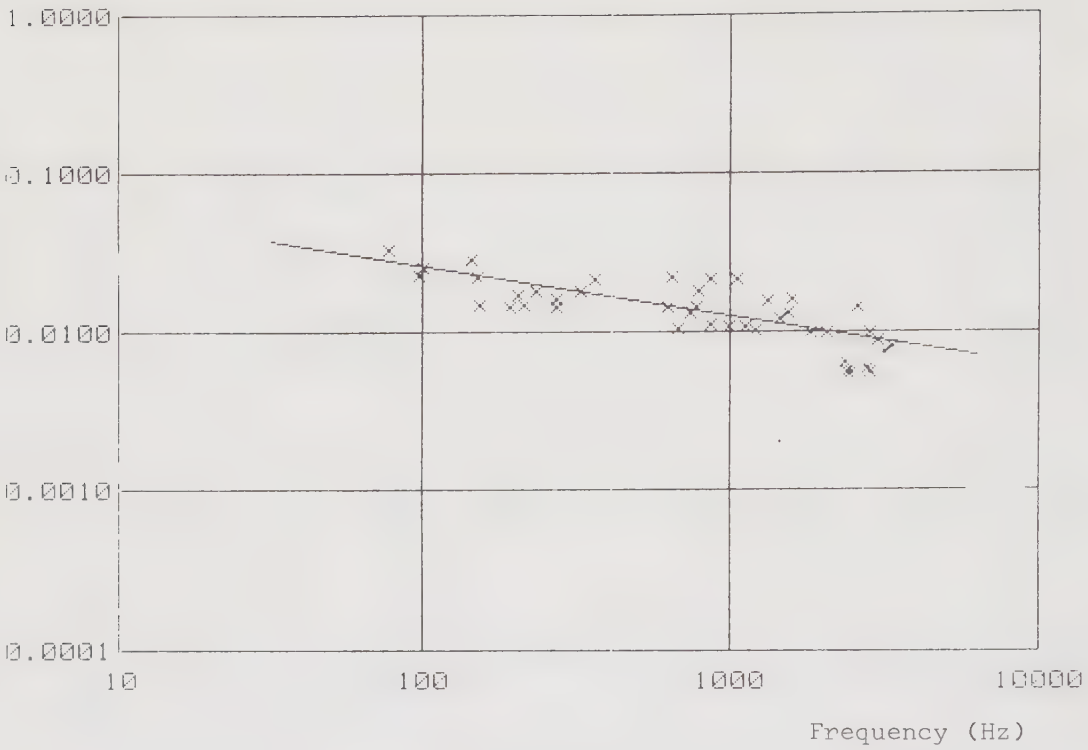
Freq. η

100	0.1552
125	0.1263
160	0.1005
200	0.0818
250	0.0665
315	0.0537
400	0.0431
500	0.0350
630	0.0283
800	0.0227
1000	0.0184
1250	0.0150
1600	0.0119
2000	0.0097
2500	0.0079
3150	0.0064

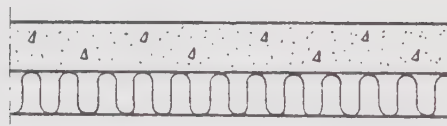
Loss factor for an 80 mm concrete slab
on 80 mm of cellular plastic (A).

III.2

Loss factor η



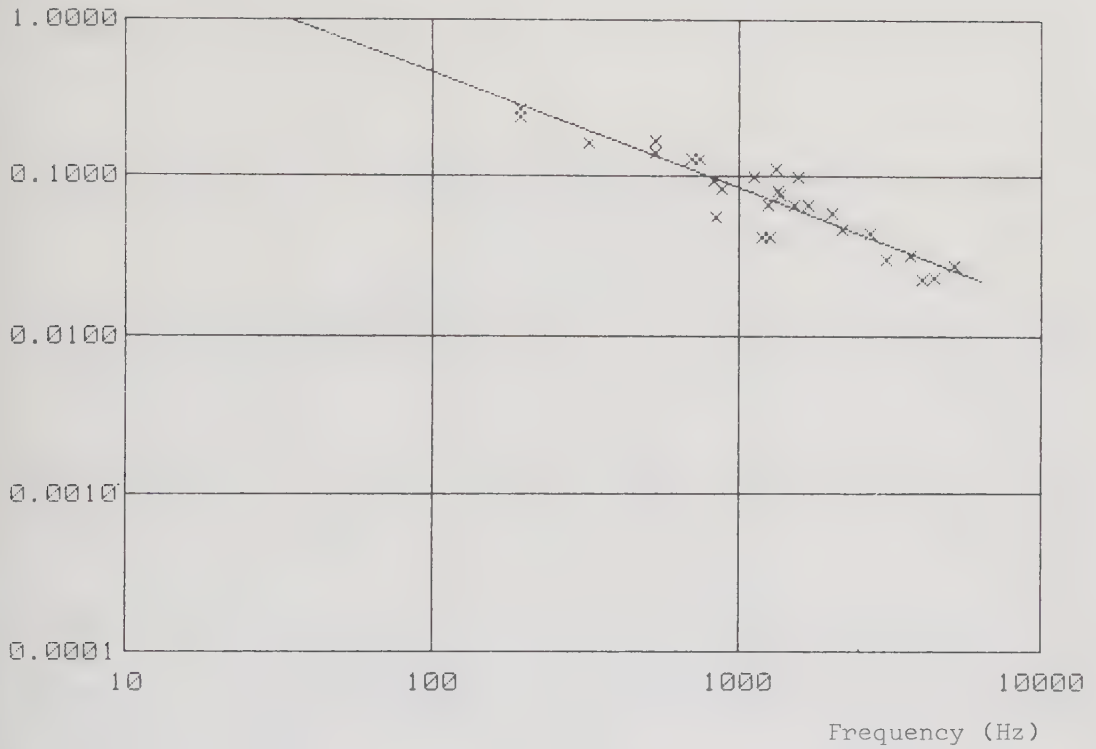
Freq.	η
100	0.0260
125	0.0242
160	0.0224
200	0.0208
250	0.0194
315	0.0180
400	0.0167
500	0.0155
630	0.0144
800	0.0134
1000	0.0125
1250	0.0116
1600	0.0107
2000	0.0100
2500	0.0093
3150	0.0086



Loss factor for an 80 mm concrete slab
on 70 mm of mineral wool (B).

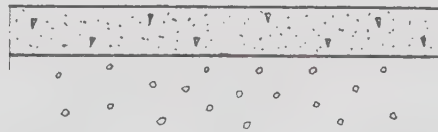
III.3

Loss factor η

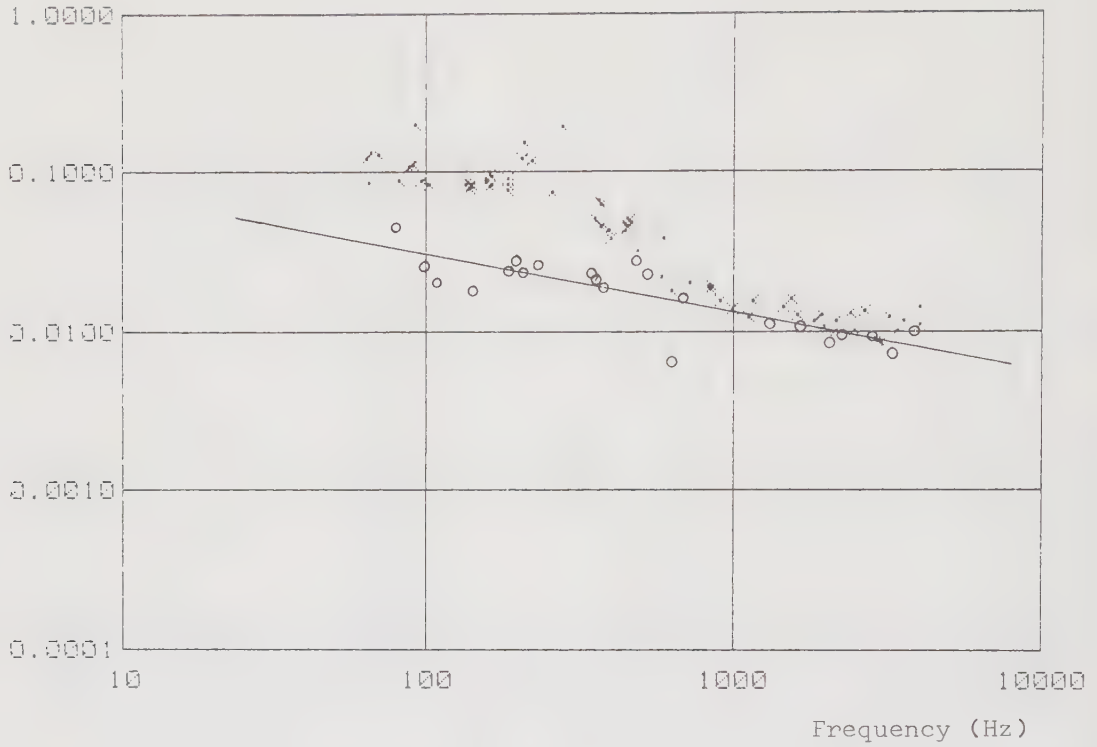


Freq. η

100	0.4576
125	0.3892
160	0.3253
200	0.2766
250	0.2352
315	0.1989
400	0.1672
500	0.1422
630	0.1202
800	0.1010
1000	0.0859
1250	0.0731
1600	0.0611
2000	0.0519
2500	0.0442
3150	0.0373



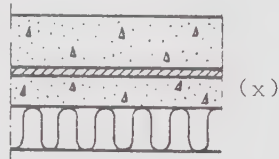
Loss factor for an 80 mm concrete slab on expanded clay (Leca).

Loss factor η 

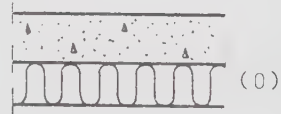
Loss factor for an 80 mm concrete slab
on two different foundations:

Foundation: Mineral wool (70 mm,
2 x 70 mm respectively)

System of
damping : Concrete tiles (50 mm),
Asfaboard (14 mm)

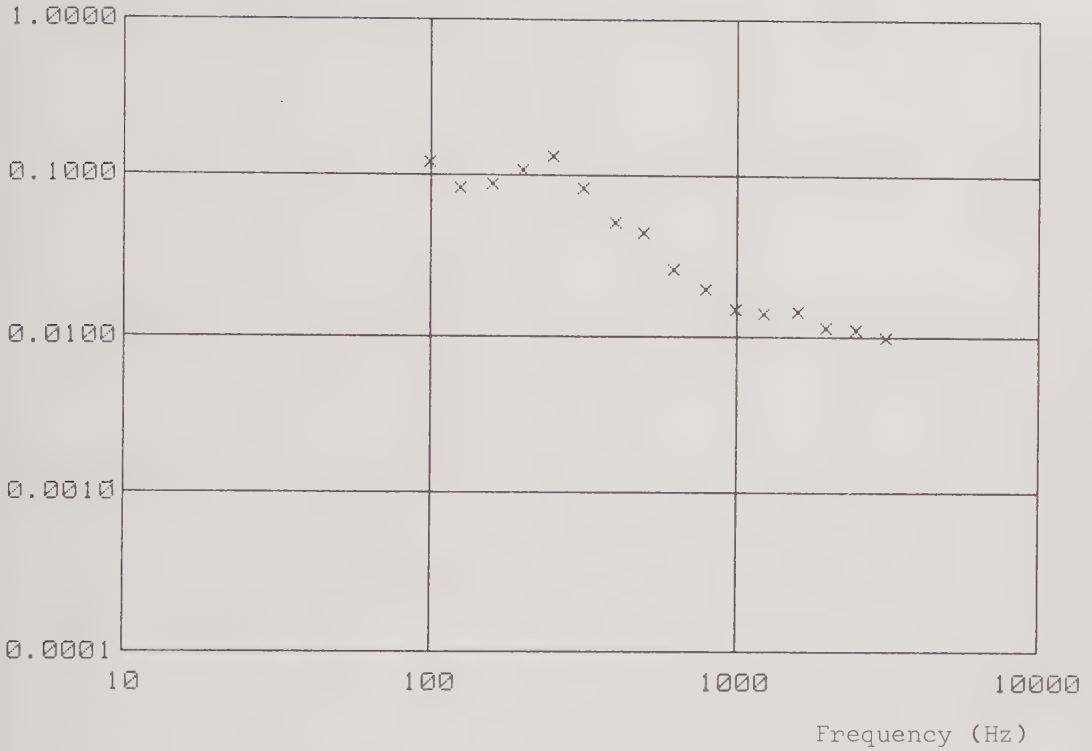


Foundation: Mineral wool (70 mm)



III.5

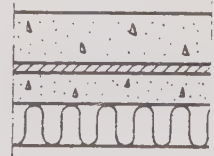
Loss factor η



Freq. η

Loss factor for an 80 mm concrete slab on the following foundation:

Foundation: Mineral wool (70 mm,
2 x 70 mm respectively)
System of
damping : Concrete tiles (50 mm),
Asfaboard (14 mm)



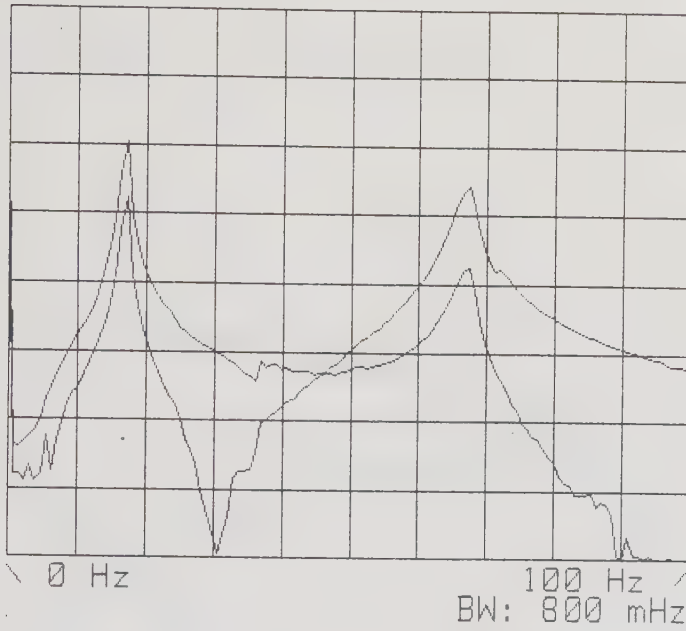
Average values from III.4

APPENDIX IV

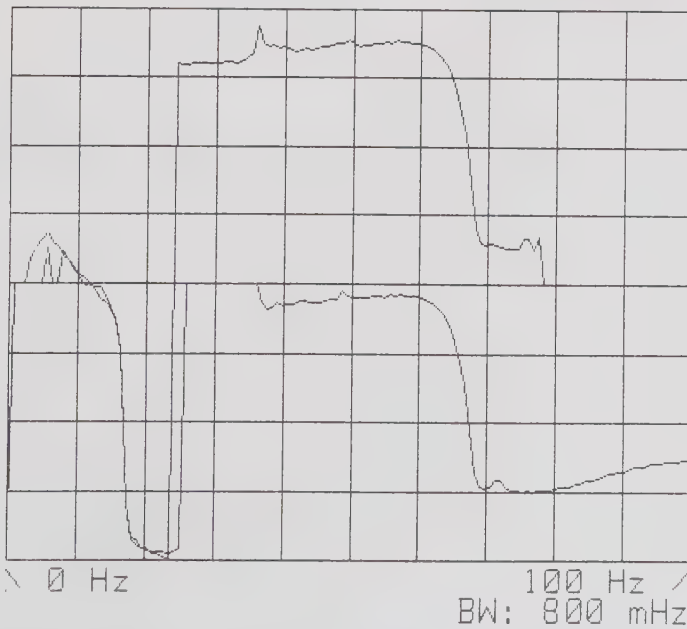
APPENDIX IV Different measurement results

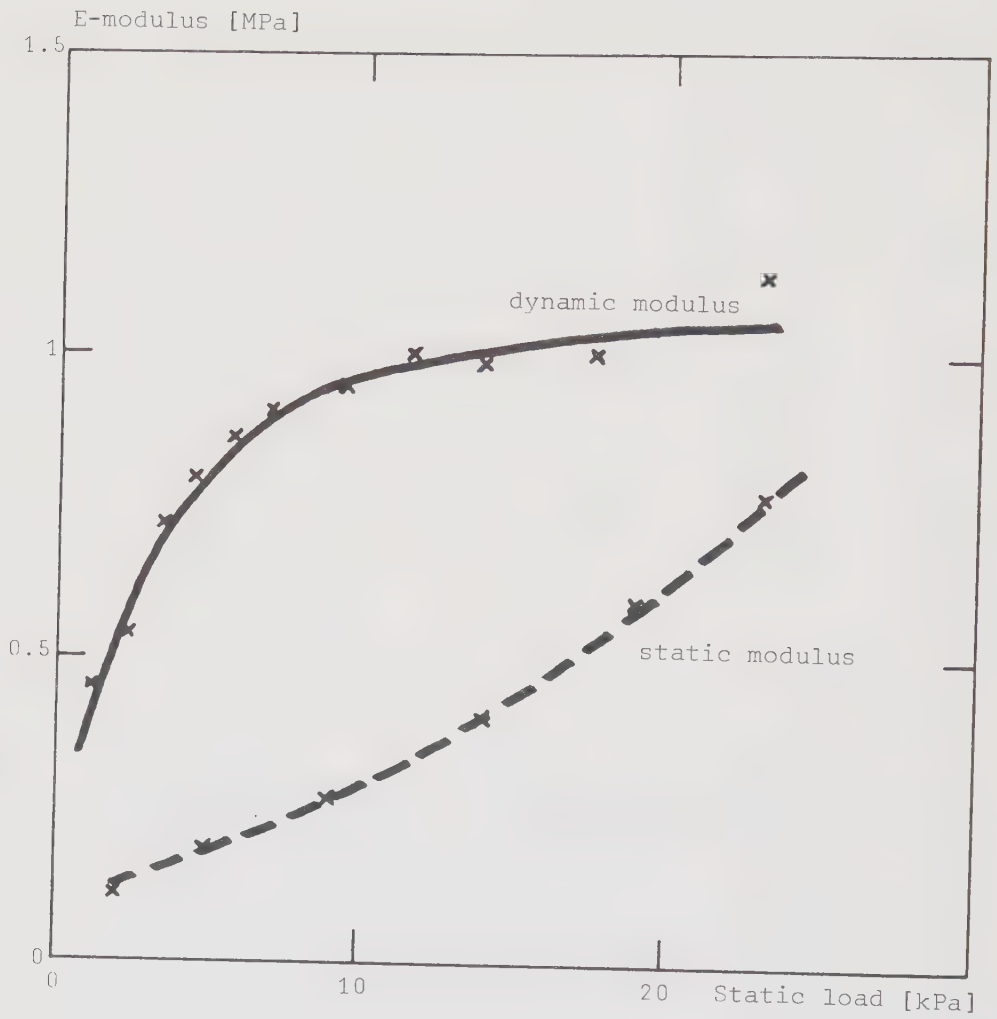
- IV.1 Typical curves from the measurement of the E-modulus.
- IV.2 E-modules for mineral wool at different static loads
 - .1 70 mm Rockwool (150 kg/m^3)
 - .2 50 mm Rockwool (150 kg/m^3)
- IV.3 Loss factor for mineral wool at different static loads.

CH A: - 30dBV FS 10dB/DIV
CH B: - 30dBV FS 10dB/DIV



CH A: 0* CENTER 50*/DIV
CH B: 0* CENTER 50*/DIV





70 mm Rockwool (150 kg/m^3)

